

Compelling in the Shadow of Holding Power*

Petros G. Sekeris[†]

*I would like to express my gratitude to Luc Lauwers, Fabrice Valognes, Jean-Philippe Platteau, Ana Mauleon, Vincent Vannetelbosch, Gilles Grandjean as well as to all Namur Seminars participants for their helpful comments.

[†]CRED (Center of Research in the Economics of Development), University of Namur, Rempart de la Vierge 8, B-5000 Namur, Belgium. e-mail: petros.sekeris@fundp.ac.be, Tel: +32 (0)494 483987, Fax: +32(0)81 72 48 40

Abstract

Power, defined as the ability to longer sustain a mutually damaging situation, determines both the outcome of the game and the way this outcome is reached. In our model, inspired from the Theory of Moves (Brams, 1994), two agents, each facing two choices at their respective decision nodes, play a sequential game over an infinite time horizon. We show that the player who is most able to incur losses -the power wielder- imposes on his opponent the strategy he wants him to adopt, the latter finding himself forced to choose between complying and being punished. These equilibrium strategies are proved to be subgame perfect and unique. In most game configurations, the power wielder can even decide the identity of the endogenously determined first mover. To make the link with a widely analyzed real world conflict situation, we apply our model to the Cuban Missile Crisis.

Keywords: Non-Cooperative Game, Holding Power, Threats Credibility, Subgame Perfect Equilibrium.

1 Introduction

The 'game of chicken' is the name retained in the literature for designating all situations where two opponents are involved in a fatal game and each hopes the other first chickens out given that the status quo is mutually damaging. Consequently, although both competitors find the on-going confrontation distressing, we may observe, in the real world, none receding as a consequence of each other's respective belief it is him that can longer endure this harmful state of affairs. The object of the exercise then is to find out who has the highest *holding power* (Kilgour and Zagare, 1987; Brams, 1994). The aim of our analysis is to understand how such disputes can be settled by altering the static game under consideration, a game which, nevertheless, gives a taste of the conflictual situations we intend to settle. Indeed, the motivation of this paper is to shed some light on the concept of power defined as *the ability of an agent to coerce another to follow some specific actions despite the potential resistance of the latter, which does not materialize as long as he acknowledges his power inferiority*. A central concern, therefore, is to correctly set the model's framework so that the players' respective power emerges from their preferences alone rather than from some *ad hoc* advantage. To this end, we construct a model where the first mover is endogenously determined, and the players who derive the payoffs associated with a cell of the payoff matrix at each time period take actions sequentially over an infinite time horizon¹. This said, we are modifying

¹A complete justification of the structure of the game could prove quite extensive. In brief, however, it goes as follows. Setting exogenously the first mover gives an arbitrary advantage

the strategic considerations of any payoff matrix under study. As an example, the payoffs derived from the players from the various outcomes during the Cuban Missile Crisis of 1962 are well illustrated by the game of chicken (Brams, 1985), but when applying our framework to this game, the players have strategy sets different from those which are typically assumed in the related theory (i.e. simultaneous, or finite-stage sequential resolution).

Our main challenge is to show that the players involved in conflictual situations make credible threats based on the power differentials that compel their opponent to immediately yield to their demands. On top of our contribution to the concepts of power and of credible threats, we also distinguish situations where the players' relative *power* even determines with a unit probability the identity of the first mover. Lastly, by the very construction of the model, rather than simply pointing at the equilibrium outcome, we provide a realistic explanation of the way a particular outcome is reached - be it an agreement, a conflict, or any other situation.

We begin by giving a simple example of the model's working; we then explore the concepts of power and of threat credibility; we continue with the presentation of the equilibrium analysis; and, in a last section, the model is applied to first mover à la Stackelberg, or second mover advantage, depending on the situations), while the simultaneous play forbids Luce and Raiffa's (1957) *power strategy* of putting another face with a *fait accompli*. Moreover, setting a last time period (terminal node) constrains the players' actions by turning some otherwise credible into incredible ones, and vice-versa. Lastly, it confers some players an exogenous power as the comparison of the dictatorial pie sharing with Rubinstein's (1982) model makes obvious.

the Cuban Missile Crisis.

An Example

This model owes its inspiration to the *Theory of Moves* of Steven Brams (1994) who aggregated in a single work the results of a series of previous articles (Brams and Wittman, 1981; Brams and Hessel, 1983, 1984; Kilgour and Zagare, 1987). The features of ToM that have attracted our attention are those that enable it to describe 2×2 games (in normal form) where a cell of the matrix is assigned the initial state, and the players can sequentially move inside the matrix by modifying their actions. The model we devise follows the same logic of ‘moving’ inside the payoff matrix from an initial state, but, the structure of the game proposed by Brams has weaknesses (see Langlois, 1991; Gilboa, 1995; Stone, 2001), that we overcome. The following example gives a first glimpse of our model’s working.

	<i>L</i>	<i>R</i>
<i>T</i>	A= {2, 4}	D= {4, 2}
<i>B</i>	B= {1, 1}	C= {3, 3}

Figure 1: A simple example.

Consider the matrix displayed in figure 1 and assume the initial state is $(2; 4)$. At each stage of the game the individuals receive a (stage-)payoff, $g_i(\cdot)$, associated with the state the game is positioned at (in $t = 0$ Row (*R*) player receives a payoff of $g_R = 2$ and Column (*C*) player gets $g_C = 4$). The two

players exchange cheap talk on the actions they would pick in the first stage of the game if they were to decide first. On the basis of this information, the first mover is endogenously determined as follows. If one player wants to move when the other does not, the former has precedence over the latter meaning the player wanting to move is assigned the first mover identity. When both players want to stay *or* to move, the first mover is determined at random with the probability of each occupying the initial node being of $\frac{1}{2}$. Once the first mover has been determined, the players sequentially take the actions of staying (*s*) or moving (*m*) in an infinite stage game whose time horizon is infinite. Each player will choose the strategy that maximizes the discounted present value of the whole sequence of payoffs he shall receive. The utility of player i ($=\{R,C\}$) can thus be written as:

$$U_i = \sum_{t=0}^{\infty} \delta_i^t g_i(x_t) \quad (1)$$

with x_t standing for the state the game is at in stage/time t .

With the elements described up to now we are already fully equipped to find the equilibrium of the game. Player C is at his most preferred state, while player R is not. The latter would like player C to move from (2; 4) to (4; 2) when, *à priori*, such an action seems irrational on C 's part. We argue that it can be rational for player C to move away from his most preferred outcome/state to the extent player R has the *power* to compel him, power which is but the ability to carry a credible threat that, if executed, would be more detrimental to player C than his moving away towards state (4; 2). In the present example, the

*threat state*² is state (1; 1) since no player would like the game to remain there. Player R 's threat of moving from (2; 4) to the threat state is credible *if* he has the ability to remain at it longer than his opponent. More precisely, imagine that player R does moves to (1; 1); and that he is then better off if he remains e_R time periods at the threat state, when the game then gradually moves to (4; 2) where a stabilization occurs, instead of moving immediately back to (2; 4) and the game remaining there. For player C , e_C then stands for the maximal number of periods he would be willing to remain at the threat state, in order that, afterwards, the game stabilizes at (2; 4) rather than at (4; 2). Once e_R and e_C have been computed, we can determine the stronger player and there-by the *equilibrium state of the game*. Suppose player R is the power wielder, we shall show that R 's threat to remain at the threat state is credible, whence in the signalling game player R signals he shall not move first and player C 's optimal action is to signal he will move. Indeed, player C being conscious of the threat's credibility, is better off by moving first rather than seeing the game pass through states (1; 1) and (3; 3) before reaching the equilibrium state³. Hence, player C is assigned at the initial node, and immediately moves from (2; 4) to (4; 2) where no player has an incentive to deviate. The respective utilities of the players are:

$$U_R^* = \delta_R^0 g_R(2; 4) + \frac{\delta_R}{1 - \delta_R} \cdot g_R(4; 2)$$

$$U_C^* = \delta_C^0 g_C(2; 4) + \frac{\delta_C}{1 - \delta_C} \cdot g_C(4; 2)$$

²The required definitions are provided in the next section.

³Notice that for this to be true we shall in a later section introduce a further assumption that reduces the class of games under study.

This example brings out the importance of the threats' credibility in the determination of an individual's power. Let us, however, consider more fully the concept of power, before reviewing what makes a threat credible.

The Concept of Power and the Credibility of Threats

Kilgour and Zagare (1987) initially define *Holding Power* as '*the ability of one player in a sequential game to absorb the costs of staying at a position (which may perhaps be Pareto-inferior) longer than his opponent*' (1987: 92).

And although the definition of the concept of power that we have given above is broader, since it takes into account *any* mechanism by which a player may coerce another to act in a specific manner and is not limited to the consideration of the 'holding ability', the present paper deals with *holding power* alone. Note however that our definition is but one way to interpret power. A general definition of the concept one could hardly object to is the one given by Max Weber :

Power (Macht) is the probability that one actor within a social relationship will be in a position to carry on his own will *despite resistance*, regardless of the basis on which this probability rests [italics added] (1964: 152)

The central idea of this definition is the ability to compel someone to act as you wish him to do when his will differs from yours, divergence of desires that may imply resistance. It is the standpoint of Dahl (1957), Hirshleifer (1991), and also of Bowles (2004) with whom we agree, going even a step further than

they do. In the framework of perfect information in which we are working, the resistance could at best be *potential*. This viewpoint is shared, among others by Hirshleifer (1991) and Goemans (2000). Indeed, citing Hirshleifer:

Cooperation, with a few obvious exceptions, occurs only in the shadow of conflict (2001: 11)

The eminent theorist of conflicts Thomas Schelling (1960) emphasizes the distinction between the threat to use force and its actual application. In a passage with which Machiavelli would have undoubtedly concurred, Schelling explains that although countries have an interest in avoiding armed conflict, the ‘threat of violence is continually on call’. Hence, the good strategist would prevent the occurrence of war through the ‘skillful *nonuse* of military forces’. We follow this line of reasoning by showing that - putting, of course, aside those obvious cases Hirshleifer refers to- even though the agents cooperate immediately we cannot deduce the absence of an underlying conflict of interest that has been resolved in favour of the stronger participant. The hidden dispute, when such a disagreement exists, is settled because, although both individuals would be willing to intimidate each other, only the power wielder is in a position to convey a credible threat. Thus arises the question of the threats’ credibility; since following Schelling’s (1960) definition of a threat, the threatener would manifestly prefer not execute the threat if the contingency to which the threat is associated did occur. Kilgour and Zagare (1987) bypass the question by setting the ‘holding power’ wielder exogenously⁴. In their exposition

⁴Notice that Brams and Hessel (1983) analyze the closely related concept of *staying power*

of the *Perfect Deterrence Theory* (Kilgour and Zagare, 2000), however, those same authors recognize the essentiality of the threats' credibility in the different, though closely related issue of deterrence. For a threat to be credible, they require that 'an actor prefers to execute a threat when the anticipated worth of doing so exceeds the anticipated worth of failing to do so. Otherwise, the threat is irrational and, hence, incredible' (Kilgour and Zagare, 2000:68). In game theoretical language, a threat is credible if the players' equilibrium strategies are subgame perfect, which simply implies that rational agents optimize at *all* decision nodes, including the terminal ones. But in that case no threat could be credible at any terminal node since, by definition, a threat is not the best short run alternative of the decision maker. Gauthier (1984) fails to understand this and tries to find arguments making the threats credible, eventually leading him to obvious contradictions. But his intuition rightly tells him that something else must be at stake that makes threats credible. According to our model, the credibility emerges because of the infinite time horizon which rules out the existence of a terminal node. In that case, the use of the Subgame Perfect Nash Equilibrium may constitute a necessary but tough path to follow.

by equally exogenously fixing the power wielder. In Brams and Hessel (1984), the concept of *threat power* is being developed, a concept close to the one of *holding power*. In this latter paper non-credible threats are not considered but the rules of the game (the rules of the ToM) are quite peculiar and they differ from ours.

2 The Model

Two players, C and R first play a signaling game Γ_s that determines the first mover of the infinite-stage game Γ .

More precisely, $\Gamma_s = \langle N = \{C, R\}, (A^{-1} = \{s, m\} \times \{s, m\}), (h^0 = x^0), O \rangle$, where A^{-1} is the action space, h^0 , the initial history of the game that stands for the (exogenously given) initial *state* of the game and O is the playing order which behaves as follows:

$$O = \begin{cases} \text{if } a_R^{-1} = a_C^{-1} = \{s, m\}, O = (\frac{1}{2}, \frac{1}{2}) \\ \text{if } a_R^{-1} = m \text{ and } a_C^{-1} = s, O = (1, 0) \\ \text{if } a_R^{-1} = s \text{ and } a_C^{-1} = m, O = (0, 1) \end{cases}$$

where $O = (p, 1-p)$ implies that R is the first mover with probability p and C is the first mover with probability $(1-p)$. Therefore, each player signals whether he wants to be the first mover or not, and, either an agreement is reached, or a random device assigns a probability of $\frac{1}{2}$ to either player being the first mover.

Once the playing order has been determined, we move on to the main game $\Gamma = \langle N, (A^t), (O), (H^t), (x^t), (g), (\delta), (u) \rangle$. That is, depending on the game's history, h^t and the playing order, O , the players take actions, a^t , that determine the state of the game in time t , x^t , which gives the players a payoff, g , that is discounted at a rate δ such that their overall utility is given by u .

The *actions* of the players are constrained by the moving order in the following way; when R is the first mover, $A^t = \{s, m\} \times \{s\}$ when t is odd, and $A^t = \{s\} \times \{s, m\}$ when t is even, while if C is the first mover the opposite holds.

Next, the *state* reached in stage t , x^t is such that : $x^t = (x|x \in \{A, B, C, D\}$
, $x = x^0 \times \prod_{\tau=1}^t a^\tau$), where (A, B, C, D) are the four potential states that make
up the four cells of a 2×2 matrix, and that are such that no player is indifferent
between two distinct cells.

The *payoff*, $g : x^t \mapsto \mathbb{R}^2$, gives each player the utility associated with every
state.

Thus, at each stage t of the game Γ , the players receive the corresponding
payoff, (g_R^t, g_C^t) . The overall *utility* derived by each player is given by:

$$u_i = \sum_{t=1}^{\infty} \left(\delta^t g_i(x^t) \right) \quad (2)$$

meaning that the players act according to the time impatience hypothesis⁵. The
whole game is summarized in figure 2.

Before carrying-on with the equilibrium analysis some further definitions and
clarifications are necessary.

Cycling : By cycling we mean that each time it is a player's turn to take a
decision, he picks m up to the moment we return to the outcome from
which the cycling was initiated. A *single* cycle is defined as:

$$a_i^t = a_{-i}^{t+1} = a_i^{t+2} = a_{-i}^{t+3} = m \Rightarrow x^{t+4} = x^t \neq x^{t+1} \neq x^{t+2} \neq x^{t+3}$$

⁵Provided we are going to show that in most 2×2 game configurations the game will
stabilize at a particular state such that from some t on, $g^t = g^{t+1} = \dots g^\infty$, the time
impatience hypothesis is tantamount to saying that the players value the *way* this stable state
is reached, in addition to the final outcome itself.

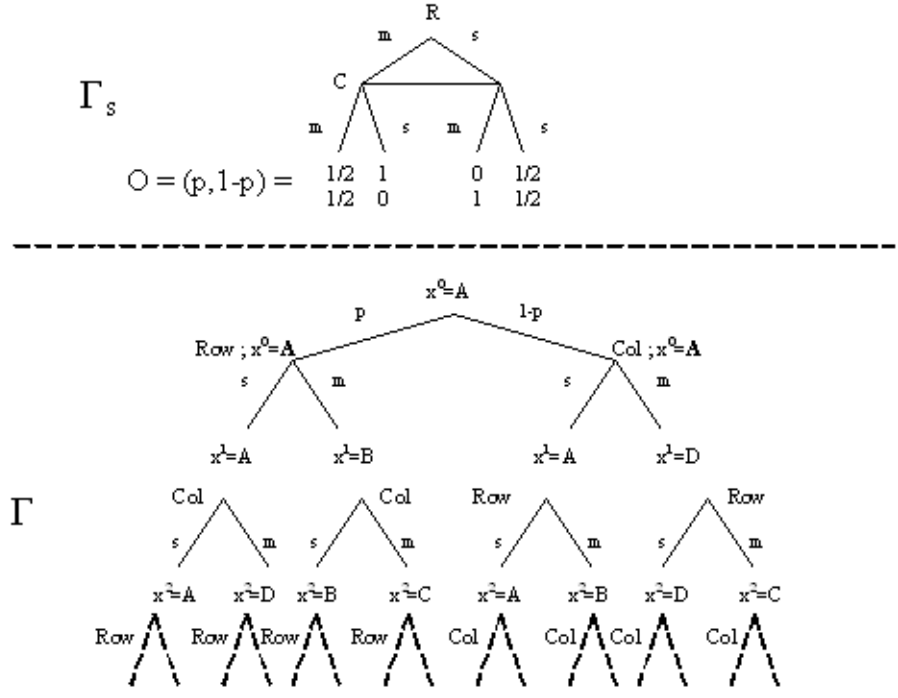


Figure 2: The Game

Threat state : A *threat state* x_ϑ is such that the following two conditions must be satisfied:

i) It must be Pareto Inferior to some other state,

ii) $g_{-i}(x^{\tau+1}(a_i^\tau = m, a_{-i}^\tau = s; x^\tau = x_\vartheta)) >$

$$g_{-i}(x^{\tau+1}(a_i^\tau = s, a_{-i}^\tau = m; x^\tau = x_\vartheta))$$

, both when $i = C$ and $i = R$

The *power* of a player determines his ability to credibly threaten his oppo-

ment. We begin therefore by defining the power variable e_i :

Power variable : Player i 's power variable e_i , measures the maximal number of periods agent i would remain at the threat state for his opponent to be the first to play m , though it would be in i 's interest to play m if $\{-i\}$ never played m . From now on if $\text{trunc}(e_i) > \text{trunc}(e_{-i})$, we shall call player i the **power wielder**⁶.

3 Equilibrium Analysis

In this section we provide the Nash and Subgame Perfect equilibrium existence proofs for different game configurations. As a preamble we provide some definitions of equilibrium concepts as well as a categorization of the 2×2 games. Afterwards we proceed with the presentation of the results.

3.1 Definitions

Each player possesses an ideal view of how the game should be played, while respecting the opponent's individual rationality constraint, in the occurrence oneself is the power wielder. This optimal playing sequence may be the same for both players. However, when this is not so, the disagreement is resolved through a comparison of the respective players' power variables. Since the weak player's optimal playing sequence will never be realized we use the term

⁶Throughout this paper we denote the truncation of player i 's power variable, $\text{trunc}(e_i)$, by e_i in order to avoid too heavy notations.

potential to underline the fact that with a different power relation it would have been enacted:

Potential Equilibrium State (PES) : A PES for player i , denoted by x_{PES_i} ,

must satisfy the following conditions:

i) $\exists(x|x \neq x_{PES_i}, g_{-i}(x) < g_{-i}(x_{PES_i}))$ {I.R. for player $-i$ }

ii) Given the initial state, x^0 , there always exists a playing sequence, (x^τ) ,

where no deviation occurs from x_{PES_i} once reached, such that for any

alternative playing sequence, $(x^{\tau'})$, where no deviation occurs from x_k

once reached, $u_i(x^t, \dots, x_{PES_i}, x_{PES_i}, \dots) > u_i(x^t, \dots, x_k, x_k, \dots)$ for any

$x^k \neq x_{PES_i}$.

{I.R. for player i }

The term *potential* stands for the possibility that this state is reached in a stable manner. When this occurs, we have an *equilibrium state*:

Equilibrium State A state is said to be the Equilibrium state if there exists

a finite τ such that the realized playing sequence from τ on is given by:

$$x^\tau = x^{\tau+1} = x^{\tau+2} \dots = x^\infty$$

Finally, we must introduce a last assumption on the model that narrows the scope of analysis because the aim of the present paper is to focus on the concept of *holding power* and we therefore defer the one of *cycling power* to a later work.

Non Cyclic Preferences The players' preferences are non-cyclic, in the sense

that:

$$\sum_{t=\tau}^{\tau+3} \delta_i^t g_i(x_{PES_\zeta}) > \sum_{t=\tau}^{\tau+3} \delta_i^t g_i(x_t) \left\{ \begin{array}{l} \text{for any finite } \tau \\ i = \{R; C\} \\ \text{for both } \zeta = i \text{ or } j \\ x_t \neq x_{t-1}, \forall t > \tau \end{array} \right.$$

This assumption precludes the kind of preferences that could make the player prefer cycling forever rather than observe the game stabilize at some moment at his preferred PES.

3.2 Categorization

As stated above, although the framework we have developed is flexible enough to deal with any 2×2 game, we only focus on the subset involving *holding power*. We nevertheless present the whole classification of the game configurations so as to enable the reader to keep track of what is done in the present work, and of what shall be attempted in a future one. A disadvantage of our approach is that the categorization is at times dubious because some game configurations fall in more than a single group depending on the players' discount rates, δ_i ; the categorization consists in a combination of payoff matrices and discount rates⁷.

No Conflict Games (NCGs) : A No Conflict Game is such that if either player was to decide the optimal strategies of both players, the result would be the same. (game 5.A. in the appendix)

⁷In the appendix we present a payoff configuration for each class of games.

Minor Conflict Games (MCGs) : A Minor Conflict Game is a game possessing no threat state, in which there is at least one *PES*. (game 5.B. in the appendix)

Partial Conflict Games (PCGs) : Any game where a threat state exists and both players have the same PES is a PCG. (game 6.A. in the appendix)

Although the players agree on the state where the game will stabilize, when $x^\tau = x^\vartheta$ both players desire their opponent to be the first to move rather than themselves.

Staying Conflict Games (SCGs) : Any game where a threat state exists and the players have distinct PESs is a SCG. (game 6.B. in the appendix)

Cycling Conflict Games (CCGs) : Any game Γ not belonging to any of the above categories. (game 7 in the appendix)

We now proceed with the analysis of those game configurations that we have announced to be the focus of this paper. We shall analyze the PCGs and SCGs since both categories involve a *holding power* potential contest. The NCGs as well as the MCGs presenting but little interest we decide not to deal with them, while the CCGs have been relegated to a future work.

3.3 Equilibrium Analysis for PCGs

Before proceeding with the equilibrium analysis *per se*, a general feature of the players' optimal behaviour needs to be underlined because of its crucial role in the subsequent developments:

Proposition 1 *If it is optimal for some finite t for a player i to take an action a_i^t , then, for whichever τ , it must be that if $x^t = x^\tau$, $a_i^t = a_i^\tau$ when i is the decision maker in both t and τ .*

for the proof see appendix.

Turning to the equilibrium analysis, a natural candidate equilibrium concept is the Nash Equilibrium. We stressed, however, the weakness of the NE since it allows for non-credible threats, while the whole notion of power crucially depends on the capacity and willingness to execute a threat. Therefore, since the players do not optimize outside the equilibrium path, there is an infinity of NE. Thence we look up for the Subgame Perfect Nash Equilibrium (*SPNE*) which is shown to be unique. The overall game being decomposed into Γ_s and Γ , when looking for the SPNE we naturally apply backwards induction and determine who effectively endorses the first mover role after having solved Γ when either player is the first mover.

The following proposition describes the players' optimal strategies when the first mover is given:

Proposition 2 *Provided a particular player gives in at x_ϑ , when in stage t the history of a Partial Conflict Game is such that $x^t = x_{PES}$, no player will ever move away from that state (equilibrium state). Moreover, should the game begin at any state k such that $x_k \neq x_{PES}$, and $x_k \neq x_\vartheta$, the players will successively play m up to when x_{PES} is reached.*

The proof being cumbersome, we present it in the appendix.

Thus, pursuing backwardsly the analysis, we endogenously determine the game's first mover. It is at this stage of the game that the equilibrium refinement we have selected becomes meaningful.

Defining a threat as incredible if it involves a strategy such that a player i is willing to remain at the threat state for t time periods, with $t > e_i$, we can state the following proposition:

Proposition 3 *The set of NE allows for non credible threats, while the set of SPNE does not.*

for the proof, see appendix.

Notice from the above proposition that we have not excluded the possibility of Nash Equilibria featuring credible threats but that do not belong to the set of SPNE. This will often be the case since, as we are to show, the set of SPNE is a singleton while the set of NE featuring credible threats is not.

Corrolary 1 *If $e_i \neq e_j$, the SPNE equilibrium strategies are unique.*

The proof is straightforward; since at the SPNE the threats must be credible, if $e_R > e_C$, the first mover in Γ_s must be player C (i.e. $a^{-1} = (s, m)$). The respective *strategies* of the players must then be such that out of the equilibrium path the threat's credibility is respected. More precisely, in case C came to deviate from the m strategy so that $x^1 = x_\vartheta$, it would still be optimal for R not to move. But, this is then true for any t , and, since the strategies for when the first mover determination has been resolved are uniquely defined, we are done. ■

Corrolary 2 *If $e_i = e_j$, the SPNE equilibrium strategies are unique up to a move from nature.*

We are now able to state the two following propositions:

Proposition 4 *In any PCG with $e_R = e_C$, the randomly determined first mover will play m , and the unique PES is reached sequentially as an Equilibrium State.*

Proposition 5 *In any PCG with $e_R \neq e_C$, there exists a unique SPE such that: (i) if $x^0 = x_{PES}$, both players pick s forever, (ii) if $x^0 = x_\vartheta$, the weakest player immediately gives in, and, afterwards, the agents sequentially pick m until x_{PES} is reached, and, (iii) if $x^0 \neq x_\vartheta$ and $x^0 \neq x_{PES}$, the commonly agreed first mover plays m and afterwards the agents sequentially pick m until the PES is reached as an Equilibrium State.*

It is interesting at this stage of the analysis to backtrack to discuss an interesting feature of the Nash Equilibria in this game.

That the NE are multiple (and even infinite because of the infinite time horizon) is straightforward since any non-credible threat is permitted. The key observation then is that there are NE featuring credible threats that fail nevertheless to be subgame perfect. Those results are summarized in figures 3 and 3'.

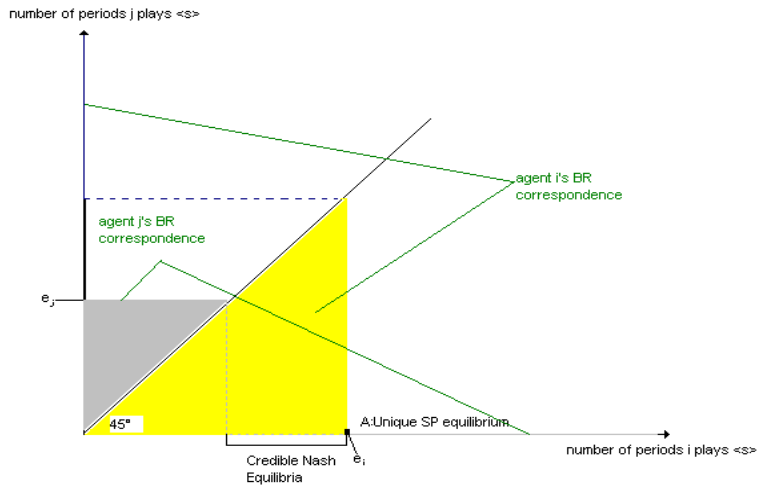


Figure 3: Credibility of threats and first mover

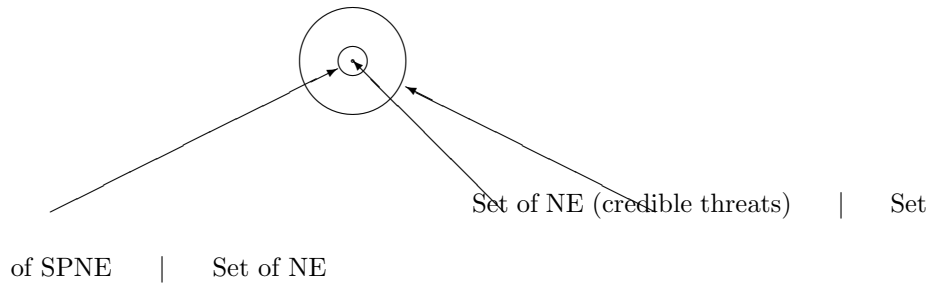


Figure 3': Comparison of the Equilibria Sets

3.4 Equilibrium Analysis for SCGs

Just as in the *PCGs*, We shall show that there is, up to nature's move, a unique SPNE for any *SCG* such that the *PES* of the power wielder is always reached as an Equilibrium State. The proof is decomposed in two parts and the proofs are presented in the appendix.

Proposition 6 *When assuming player i never gives in at x_ϑ , there is a unique SPE that drives the game onto PES_{-i} , and such that no deviation from that state occurs ever after.*

When either player is seen as having an infinite power, as long as his opponent's power variable is not infinite, it is the latter that will always give in at the threat state by a reasoning identical to the proof of proposition 6.

What should next be shown in order that we have an equilibrium where the weak player always gives in at x_ϑ is that, given proposition 6, it is optimal for the power wielder to never give in at x_ϑ . For the same reason as above, we look for the *SPNE* of this game and can make use of proposition 3 so that we only need to be concerned with strategies involving credible threats. Following an equivalent reasoning to the one used in the previous section and relying upon proposition 6, we can establish the parallel results to propositions 4 and 5 for the *PCGs*:

Proposition 7 *In any SCG where $e_R = e_C$, there exists a unique SPE, up to a nature's move. The equilibrium strategies are such that the randomly determined first mover gives in immediately and his opponent's *PES* is reached as*

an Equilibrium State

Proposition 8 *In any SCG where $e_R \neq e_C$, if $e_R > e_C$ and that $a^{-1} \neq (m, m)$, there exists a unique SPE such that: (i) if $x^0 = x_{PES_R}$, both players pick s forever, (ii) if $x^0 = x_\emptyset$, player C immediately gives in, and, afterwards, the agents sequentially pick m until x_{PES} is reached as an equilibrium state, and, (iii), if $x^0 \neq x_{PES_R}$ and $x^0 \neq x_\emptyset$, the first mover plays m and afterwards the agents sequentially pick m until x_{PES} is reached as an Equilibrium State. If, however, $e_R > e_C$ and $a^{-1} = (m, m)$, then nature determines the first-mover and the rest of the results hold.*

It will be noticed from proposition 8 that there is a slight difference with the equivalent statement for the *PCGs*; there may be payoff configurations that would push the weak player towards taking the first move in order to benefit in the short run from those outcomes conferring him a high one-stage utility, despite the fact that the equilibrium state will be the power wielder's *PES*.

Interestingly, we are able to preserve the singleton property of the *SPNE* set as well as the existence of *NE* not belonging to the *SPNE* set that nevertheless feature credible threats.

4 The Cuban Missile Crisis

The Cuban Missile Crisis was triggered by an attempt of the Soviet Union to acquire the ability to strike with nuclear weapons a significant part of the United States by installing intermediate-range missiles in Cuba. The U.S. had to react

to this move, as soon as it became aware of it, not only because of the threat the aforementioned weapons represented, but also because of the contestation of the power balance their installation implied. A game configuration that has been used to model this conflict is the game of chicken (Brams, 1994; Kilgour and Zagare, 2004) whose payoff matrix is displayed in figure 4. Although the preference rankings may be said to correctly reflect the concerned parties' true ones it is extremely difficult to estimate the magnitude of the payoffs, even ex-post. Therefore, let us first conduct the analysis by considering the payoffs depicted in figure 4 as just reflecting the ranking of the preferences. The game starts at $x_0 = D = (2; 4)$ when the soviets have already installed their missiles (Hard stance, H) and the U.S. has not yet reacted (Soft stance, S). There exists a threat state, $x_\vartheta = C = (1; 1)$, and there are two PES, the initial state for the soviets, and state B for the United States. At state A either player wants to deviate so as to reach his PES. The game is therefore a SCG.

		Soviet Union	
		S	H
United States	S	$A = (3, 3)$	$D = (2, 4)$
	H	$B = (4, 2)$	$C = (1, 1)$

Figure 4: The Cuban Missile Crisis.

Power variables: e_{US} and e_{SU} :

The power variable for the US satisfies:

$$\delta_{US}^0 g_{US}(C) + \sum_{t=1}^{\infty} \delta_{US}^t g_{US}(D) = \sum_{t=0}^{e_{US}} \delta_{US}^t g_{US}(C) + \sum_{t=e_{US}+1}^{\infty} \delta_{US}^t g_{US}(B)$$

which, after some basic algebra yields:

$$e_{US} = \frac{\ln\left(\frac{g_{US}(D) - g_{US}(C)}{g_{US}(B) - g_{US}(C)}\right)}{\ln(\delta_{US})}$$

while, the same computation for the *S.U.* results in:

$$e_{SU} = \frac{\ln\left(\frac{g_{SU}(B) - g_{SU}(C)}{g_{SU}(D) - g_{SU}(C)}\right)}{\ln(\delta_{SU})}$$

Before climbing up the game tree to solve the signalling game, it is interesting to observe that:

(i) The more impatient the agents are, the lower their power will be; A player whose discount factor is low is less willing to suffer the costs associated with the threat state in order to obtain a higher payoff in later periods.

(ii) The higher the utility one derives from his own PES, or, the lower the utility one derives from his opponent's PES (provided this value remains superior to $U(x_\emptyset)$), the higher will his power be.

(iii) By increasing the threat state's value the power of a player is equally enhanced; since the cost of carrying the threat gets closer to the value of giving in to the opponent's will, the agent can sustain x_\emptyset longer.

In order to determine the equilibrium strategies of the players and to be in a position to make any kind of prediction, we have to be able to compare their power variables so as to determine the players' equilibrium strategy in the signalling game. But this requires assigning the players their real utilities

as well as their true discount rates - issues that even nowadays are debatable. Given, however, the way the crisis was eventually settled, we infer the players' preferences and explain what would have happened had the players been in possession of all the relevant information on themselves, as well as on their opponent.

History records that the confrontation had to climb disproportionately high for the Russians to eventually yield. We can therefore infer that the U.S. had a higher holding power, which, depending on the payoffs and on the discount rates, gives rise to various interpretations. For simplicity we may assume the U.S. was more patient (say $\delta_{US} = 0.9$) than the Soviets ($\delta_{SU} = 0.7$)⁸ and that the values appearing in figure 4 stand for the players' cardinal utilities. Then the players' respective power would have been $\text{trunc}(e_{US}) = 10$ and $\text{trunc}(e_{SU}) = 3$. We can make direct use of proposition 8 by associating the *U.S.* with Row player, implying that the unique SPNE is such that:

- In the signalling game, $a^{-1} = (s, m)$.
- In the main game, on the equilibrium path, $a^0 = (s, m)$, $a^1 = (m, s)$, and, in $t = 2$, $x^2 = B = x_{PES_{US}}$, hence, no player deviates from it.
- Outside the equilibrium path, the *U.S.* never yields at x_ϑ while the *S.U.*

⁸We could also have assumed that because of the geographic position of Cuba, yielding to the ambition of the Soviets to grant themselves a strategic edge would have been almost as damageable to the U.S. than is the threat state. It is pointless, however, to spend time on speculations. Not being in possession of the relevant information, we chose to arbitrarily consider one potential scenario.

always does. At state A , the US play m , while at state D the SU does so.

What the analysis implies is that the Russians would not have supported more than 3 time periods at x_{ϑ} when the US could have waited much longer in order for the Soviets to leave the Cuban island. Bearing in mind that x_{ϑ} is the outcome when both players act aggressively *vis à vis* each other, the Americans were disposed to escalate the conflict more than the Russians. In conformity, therefore, with the model's predictions, the S.U. should have yielded from the very beginning.

The reality, nevertheless, proved to be quite different. The Soviets removed the missiles from Cuba and the U.S. promised not to attack Cuba and offered to the Soviets a face saving informal assurance that it would remove its (obsolete) Jupiter missiles from Turkey. But, despite the removal of the soviet missiles, Washington maintained a hard stance towards Moscow. The stable state is indeed B as predicted. Yet, according to the model, it is the Soviets that should have moved first, and only then would the U.S. harden its position by whichever means (imposition of sanctions for that intolerable act, increase in military spending...). Hence, although this model predicts correctly the final outcome, it fails to pin down the sequence that led to the equilibrium state, i.e. the equilibrium path. The S.U. did not move first, it was the U.S. that imposed a naval blockade on Cuba as a first warning. Given the non-responsiveness of the opposite side, the U.S. administration ordered an increase in military readiness. The downing, however, of an American reconnaissance aircraft over the Island of

Cuba was a sign that the Russians believed they could still maintain their initial position. The crisis had to climb quite high for the Russians to understand that their enemies would not tolerate those missiles on their doorstep. Only when the imminence of a nuclear conflagration became evident to Khrushchev did he realize the Americans would not back down. What followed was an 'ignominious retreat' and a 'severe humiliation' (Kissinger, 1994). Thus the U.S. presented a firm front to the Russians. Both the agreement not to invade Cuba and the promise over Turkey were devices for Moscow to save face. On the other hand, despite maintaining this firm stance -accompanied, it should be added, by build up of nuclear forces - it did not adopt an offensive strategy that could have triggered a new crisis⁹.

From the matrix point of view, the Americans moved first, and, once at C (the threat state) the game remained there for a certain number of periods with no side giving in. After a moment, the Russians accepted to move towards B, the equilibrium state.

The divergences between the predictions of this model and reality need to be understood in the light of the complete (and perfect) information hypothesis we have made. Either because they thought they could convince their opponents that they had the power they knew they lacked (bluff strategy), or, indeed, because they were certain of their power superiority, the Soviets believed that they could maintain state D as the equilibrium state. Whatever the motivation

⁹If the U.S. had adopted an offensive strategy we would be facing a different game, since a hard stance is quite different from an offensive posture. The U.S. in Cuba was in a way defending itself; this should not be confused with an aggressive stance.

explaining their behaviour, however, the U.S., for its part, was convinced (and rightly so) that it was the power wielder and that it had to prove it to its opponent by hardening its position. Though *ex-post* the U.S.'s actions (i.e. hardening its position) are still rational, the Russians played sub-optimally. This observation seems to credit the existence of informational asymmetries among rational players - a topic that is beyond the scope of this work. It could be argued that the present model fails to capture the essence of the conflict, i.e. the catastrophic consequences of a lack of coordination in the simultaneous-move one-shot version of the game of chicken. We do not, however, share this view. Just as Schelling's idea of brinkmanship suggests, available choices were not limited to conducting a nuclear war or not. Even if the U.S. had engaged in a war against the S.U., this conflict might have proven to be from a limited (use of conventional weaponry, military/strategic targets) up to a disastrous one (both sides unleashing their nuclear arsenal on each other), with all the conceivable intermediate levels of violence also possible.

5 Concluding Remarks

Our concern throughout this paper has been to develop a meaningful notion of power. Using the concept of *holding power*, that we further refined, we have been able to explain, by simply referring to the players' power differential, both the equilibrium state of the sequential 2×2 game, and the path leading to this (final) outcome. We have focused on those games where holding power is

effective, ruling out, therefore, what we have called ‘cyclic preferences’. Given these assumptions, we proved that, provided a power differential exists, the unique *SPNE* is such that the weak player yields immediately to his opponent’s will. Moreover, in such situations the playing order is endogenous because of the power wielder’s ability to impose both the playing sequence and the actions, in the range of the individually rational playing sequences. The same results hold true, up to a nature’s move, in the case where the players’ power variable is equal. We are thus able to explain not only why a particular path was followed but also *what did not* happen because of some (not always obvious) power relation between players who are aware of each other’s coercive abilities. Hence the statement that cooperation occurs in the shadow of power. Thus, although cooperation may be perfect, we can not preclude before analyzing the situation that behind the consensus lies a power relation which allows one agent to require his opponent’s total subordination. It is interesting to notice that, though the time and states preferences are not comparable across players, the model allows us to confront them through the players’ respective power parameters.

This model has a strong flavour of realpolitik, as is evident in our appreciation of the Cuban Missile Crisis. Although we fail to pinpoint the superpowers’ actions, we do predict the final outcome of the crisis, while believing that behind the USSR’s decision not to yield to the American demands lies an informational asymmetry.

Appendix

Proof of proposition 1

Define first a Best Response function, BR_i as:

Definition: In game Γ , the strategy $BR_i = \sigma_i = ((a_i^t(h^t))_{t=0}^\infty)$ is a best response for player i to his rival's strategy, σ_{-i} if:

$$u_i(\sigma_i, \sigma_{-i}) > u_i(\sigma'_i, \sigma_{-i})$$

for all $\sigma'_i \in \Sigma_i$, where Σ_i is the strategy set of player i .

Take $a_i^\tau, a_i^{\tau'} \in BR_i$. Then, if $\tau' \neq \tau$ is such that $x^{\tau'} = x^\tau$ and $A^{\tau'} = A^\tau$, the two subgames beginning at τ and τ' are identical. Thus, if $a_i^{\tau'} \neq a_i^\tau$, either both actions are optimal implying that without loss of generality we could impose player i picking the same action in both initial nodes of the two respective subgames ($a_i^{\tau'}$ or a_i^τ), or one of the actions does not belongs to BR_i , which implies a contradiction ■

Proof of proposition 2

Suppose that $x^\tau = x_{PES}$ and that $A^\tau = (A_R^\tau, A_C^\tau) = (\{m, s\}, s)$. Next denote by $(x)_{PES}^\tau$ the sequence beginning in $t = \tau$ such that $x^\tau = x_{PES}$ and no deviation occurs from x_{PES} . Imagine that, contrary to what the proposition states, $a_R^\tau = m$, then, it must be that $u_R((x)^\tau) > u_R((x)_{PES}^\tau)$, for some alternative playing sequence. But this means that there is some state $x_P \neq x_{PES}$ such that $g_R(x_P) > g_R(x_{PES})$.

Consider first that for some $t = \tau'$, $x^{\tau'} = x_P$, and that $a_C^{\tau'-1} = m$. Then, should $a_R^{\tau'} = s$, following proposition 1, and the fact that $g_R(x_P) > g_R(x_{PES})$, $x_P = x_{PES}$ which is a contradiction.

Should R deviate from x_{PES} , the following cases need to be considered:

i) Notice that $x^{\tau+1}(x^\tau \times (m, s)) \neq x_\emptyset$ because, otherwise, either the definition of the *PES* is violated, or R has cyclic preferences. Indeed, if $x^{\tau+1}(x^\tau \times (m, s)) = x_\emptyset$, and that R has not cyclic preferences, then it must be that R expects x_P to be reached as an equilibrium state. But then, either R will be the one taking the move that drives the game onto that state, and as explained above C moves and we have a cycling outcome, or C is the one that drives the game onto x_P . In the former case we reach a contradiction since cyclic preferences are forbidden, while in the latter $e_C = \infty$, meaning he would *never* move away from x_\emptyset .

ii) Next, if $g_R(x^{\tau+1}(x^\tau \times (m, s))) > g_R(x_{PES})$, then, $g_C(x^{\tau+1}(x^\tau \times (m, s))) \leq g_C(x^t), \forall x^t \in \{A, B, C, D\}$, otherwise x^τ could not be a *PES*. But then, given that in any *PCG* there exists a threat state, in conjunction with proposition 1, it must be that $x^{\tau+2}(x^\tau \times (m, s) \times (s, m)) = x_\emptyset$. But this in turn implies that $g_R(x^{\tau+1}) > g_R(x_\emptyset)$ since $g_R(x^{\tau+1}) > g_R(x_{PES})$. Provided R moved from x_{PES} to $x^{\tau+1}$, player $\{C\}$ knows that if he plays s in $x^{\tau+1}$, player R will not play m . Thus, $a_R^{\tau+1}(x^\tau; (m, s)) = m \Rightarrow x^{\tau+2} = x_\emptyset$. But by the definition of a *PCG*, any player is better with a playing sequence that stabilizes at the *PES* compared with the sequence that stabilizes at x_\emptyset . Therefore, it must be rational that some of the two players gives in at that suboptimal state, otherwise, this state is a *PES*, which cannot be true. Hence, $a_R^{\tau+2}(x^\tau; (m, s); (s, m)) = m$, since C will not

move back to $x_{\tau+1}$, for, his initial move away from $x_{\tau+1}$ to x_{ϑ} would have been irrational. But then, player $\{C\}$ either moves and $x^{\tau+4} = x^{\tau} = x_{PES}$, or does not. If he does, the non-cyclicity of the preferences has been violated, while if he does not, it is R 's turn to take an action. If $a_R^{\tau+4} = s$, $x^{\tau+3} = x^{\tau+4} = x^{\tau+5} \dots$, hence x^{τ} is not the PES , which cannot be true. $a_R^{\tau+4} = m$, the game moves back to x_{ϑ} where C will play s making this move towards x_{ϑ} suboptimal. Thus, when $g_R(x^{\tau+1}(x^{\tau} \times (m, s))) > g_R(x_{PES})$, $a_R^{\tau} = s$.

If $g_R(x^{\tau+1}(x^{\tau} \times (m, s))) < g_R(x_{PES})$, either we are facing a case that is symmetric to *ii*) and we are done, or $g_C(x^{\tau+1}(x^{\tau} \times (m, s))) < g_C(x_{PES})$ and an identical reasoning to the one carried in *ii*) leads to the same conclusion that $a_R^{\tau} = s$

The same reasoning can be carried when inverting the players' identities, hence we're done for the first part of the proposition. As for the second part, it is a corollary of the above proofs ■

Proof of proposition 3

To check the credibility of the players' equilibrium strategies when looking for the SPNE of this game, we need to show that a player's best response must be in accordance with his power variable for a given first mover. The reasoning for player R is the following: If $O = (1, 0)$, then if $(a_C^t)_{t=1,3,\dots}^{\tau} = s$, $a_C^{\tau+2} = m$, and $\tau \geq e_R$, then $a_R^0 = m$, while if $\tau < e_R$, $a_R^0 = s$. This follows from the computation of e_R .

Lastly, the Nash Equilibrium is such that: $BR_R(BR_C(s_R^*)) = s^*$. But then at equilibrium a player, say C , could not play s at x_{ϑ} in time t when $t \geq e_C$ since

this would violate the above. Thus, only strategies involving credible threats belong to the set of Nash Equilibria.

which implies that given the opponent's strategy σ , staying more than e_R time periods at x_ϑ violates the I.R. constraint. This reasoning warrants that no SPNE will allow for non credible threats but does not, however, precludes the players making non credible threats at a NE ■

Proof of Corrolary 2

When $e_R = e_C$, no player can coerce the other to move first. Both are aware, however, that when neither moves, $O = (1/2, 1/2)$. Moreover, suppose player R is the randomly selected first mover when neither player moves in the signaling game. Following a reasoning as Shaked and Sutton (1983), there is no 'proposal' R could make in $t = 1$ so that C gives in the next time period. Hence, the best offer R can make is s . Then, in $t = 2$, C will stay, implying that $x^0 = x_\vartheta = x^1 = x^2$. But then, applying proposition 1, $x^t = x_\vartheta, \forall t$. But then $a_R^1 = s \notin BR_R$ since an immediate deviation is individually rational. As a consequence, the player who is randomly assigned the first mover status is the one who yields, and, in knowledge of that, no player has an incentive not to play s in the signaling game. Hence, the only 'undetermination' is with respect to nature's move ■

Proof of proposition 6

Suppose that player R alone is taken to never give in at x_ϑ , then, by assumption, if $x^t = x_\vartheta$ and $A_R^t = \{s, m\}$, $a^t = (s, s)$, and if $A_R^t = \{s\}$, $a^t = (s, m)$.

Suppose next that τ is odd, that C is ‘on the move’ when t is odd, and that $x^\tau = x_{PES,R}$. If $a^\tau = (s, m)$, three distinct cases need to be considered:

i) $x^{\tau+1} = x_\vartheta$. But, by assumption then, $a^{\tau+1} = (s, s)$, $a^{\tau+2} = (s, m) \Rightarrow x^{\tau+3} = x_{PES,R}$. But then it could not be optimal for C to deviate initially from $x_{PES,R}$.

ii) $x^{\tau+1} = x_{PES,C}$. Then, $a_R^{\tau+1} = m \Rightarrow x^{\tau+2} = x_\vartheta$ by definition of the threat state. The unique reason that would then prevent player C from playing m would be that this costly move (costly because $g_C(h^{\tau+2}, (s, m)) < g_C(x_\vartheta)$, by definition of the threat state) implies that whatever the subsequent sequence of states that would be played, C would be better by remaining at x_ϑ . But then, C ’s power variable is infinite which is ruled out by assumption. Thus, $\Rightarrow a_C^{\tau+2} = m \Rightarrow a_R^{\tau+3} = m \Rightarrow x^{\tau+4} = x_{PES,R}$. Having assumed the players have non-cyclic preferences, C will not initially deviate from $x_{PES,R}$.

iii) $x^{\tau+1} \neq x_\vartheta$ and $x^{\tau+1} \neq x_{PES,C}$. This could not be so for it would contradict condition *ii)* in the definition of a threat state.

Thus C plays m at x_ϑ , once $x_{PES,R}$ is reached it is an Equilibrium State, and, at any other state the player on the move plays m . Hence the proof is done when the game begins at x_ϑ , but the same reasoning can be carried for when the game begins at any cell of the matrix■

Examples of Games Following our Categorization

	L	R
T	2, 2	1, 1
B	3, 3	4, 4
	A	

	L	R
T	1, 1	2, 3
B	3, 2	4, 4
	B	

Figure 5: A . No Conflict Game. B . Minor Conflict Game

	L	R
T	1, 1	3, 2
B	2, 3	4, 4
	A	

	L	R
T	1, 1	4, 2
B	2, 4	3, 3
	B	

Figure 6: A . Partial Conflict Game. B . Staying Conflict Game

References

- [1] S. Bowles. *Microeconomics: Behavior, Institutions and Evolution*. Princeton University Press, New York, 2004.
- [2] S. Brams. *Superpower Games*. Yale University Press, 1985.
- [3] S. Brams. *Theory of Moves*. Cambridge University Press, Cambridge, 1994.

	L	R
T	4, 2	1, 4
B	2, 3	3, 1

Figure 7: Cycling Conflict Game for very low discount rates

- [4] S. Brams and Wittman D. Nonmyopic equilibria in 2×2 games. *Conflict Management and Peace Science*, pages 39–62, 1981.
- [5] S. Brams and Hessel M. Threat power in sequential games. *International Studies Quarterly*, 28:23–44, 1984.
- [6] S. Brams and Hessel M. P. Staying power in sequential games. *Theory and Decision*, 15:279–302, 1983.
- [7] I. Gilboa. Book review of theory of moves by steven brams. *Games and Economic Behaviour*, 10:368–72, 1995.
- [8] J. Hirshleifer. The paradox of power. *Economics and Politics*, 3:177–200, 1991.
- [9] M. Kilgour and Zagare F. Holding power in sequential games. *International Interactions*, 13(2):91–114, 1987.
- [10] M. Kilgour and Zagare F. *Perfect Deterrence*. Cambridge University Press, Cambridge, 2000.
- [11] H. Kissinger. *Diplomacy*. Simon & Schuster Adult Publishing Groups, New York, 1994.
- [12] J-P. Langlois. Rational deterrence and crisis stability. *American Journal of Political Science*, 35 (4):801–832, 1991.
- [13] D. Luce and Raiffa H. *Games and Decisions: Introduction and Critical Survey*. John Wiley & Sons, New York, 1957.

- [14] A. Rubinstein. Perfect equilibrium in a bargaining game. *Econometrica*, 50(1):97–109, 1982.
- [15] T. Schelling. *The Strategy of Conflict*. Harvard University Press, Cambridge, Massachusetts, 1960.
- [16] A. Shaked and Sutton J. Involuntary unemployment as a perfect equilibrium in a bargaining model. *Econometrica*, 52(6):1351–1364, 1983.
- [17] R. W. Stone. The use and abuse of game theory in international relations. *Journal of Conflict Resolution*, 45(2):216–44, 2001.
- [18] M. Weber. *The Theory of Social and Economic Organization*. Translated and edited by A.M. Henderson and Talcott Parsons, Free Press, Glencoe, Illinois, 1964.