

Quality Choice, Sales Restriction and the Mode of Competition

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Abstract

A regulator imposing “sales restrictions” on firms competing in oligopolistic markets may enhance quality provision by these firms. Moreover, for most restrictions levels, the impact on quality selection is invariant to the mode of competition.

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1 Introduction

Many consumers’ market are subject to quality regulation. To the minimum, regulations impose as a general requirement that products should be “safe”. However, either because of various forms of externalities or more simply for paternalistic reasons, governments directly set minimum quality standards that firms must comply with. This is for instance the case in the automobile industry where fuel-economy standards must be met by producers. Actually, environmental requirements are becoming more and more pervasive in mass product industries and tend to be assimilated to quality. Child care centers must meet drastic criteria in order to obtain accreditation. More generally, many service (in particular human services) industries see their access conditioned by various forms of certification aimed at controlling for adequate professional training. Again, the aim is to ensure, by regulation, that “quality” is large enough.

The fact that quality must be ensured through regulation might seem surprising at first sight. We may indeed expect that only high quality products are apt to survive in a competitive market. However, the fact that competition is compatible with low quality products does not come as a surprise to industrial organization scholars. This is indeed a very standard result in the literature dealing with quality product differentiation. Actually, entering

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a market with a low quality product is often an optimal strategy (Crampes and Hollander (1995)). Moreover, in case of entry, it might be optimal for an incumbent to downgrade its own quality (Yong (1999)). Competition erodes profits margins; thus if quality provision is costly, quality upgrades tend to be less valuable at the margin. More generally, quality choices are governed by strategic considerations and in this respect, quality downgrades are often valuable at the margin because they relax competition, in addition to decreasing costs.

In regulated industries, ensuring quality provision has also been a concern for public authorities. In an asymmetric information context, it is well known indeed that the regulator has to mix incentives and monitoring to ensure quality provision. The regulation literature that started with Baron and Myerson (1982) has shown the inherent limits of this approach in terms of efficiency and the cost of public funds. In this context, competition has often been viewed as a way out of the dilemma. One could indeed hope that lower quality products would not resist the challenge of higher quality products entering the (now) deregulated industry. However, the accidents of privatized railways in the UK or the power crisis in California shocked the population and lead policy makers to realize that increased competition does not necessarily ensure quality provision but could actually undermine it. For instance, Kennet (1993) showed in an empirical paper that a lowering of quality initially followed the deregulation of the airline industry in the US.

Accordingly, introducing, or enhancing, competition may call for complementary public intervention aimed at ensuring quality provision. Two basic instruments have been studied in the literature: Minimum Quality Standards (MQS hereafter) and tax/subsidy schemes. Obviously, by imposing a MQS, we increase quality at the bottom of the quality range. Ronnen (1991) provides a detailed analysis of the effects on MQS and concludes that these may indeed increase welfare in addition to increase quality. However, other papers (see for instance Scarpa (1998), or Maxwell (1998)) have studied the impact of MQS and yielded more ambiguous results regarding welfare. A major drawback of the MQS is that they undermine firms' profit by intensifying price competition. Moreover, as shown by Valletti (2000), Ronnen's conclusions regarding the positive impact of MQS are not robust to the mode of competition. All in all, theoretical approaches provide at best a mixed support for MQS policies, even when monitoring costs are neglected.¹ On the other hand, empirical evidence suggests that MQS might not induce quality upgrading. For instance Chipty and Witte (1997) empirically investigate health care providers and conclude that minimum standards increase the probability that firms exit certain markets, thereby reducing competition.

Instead of regulating quality by altering directly the quality space, the regulator may provide firms with incentives to upgrade quality. Tax/subsidy schemes might be effective in

¹Notice that monitoring is mandatory in the case of an effective MQS since by definition the MQS imposes a *binding* constraint on the low quality firm. As recently argued by Glaeser and Shleifer (2001), quantity regulation might be optimal simply because it would be less costly to enforce due to smaller monitoring costs.

this respect, as shown for instance by Cremer and Thisse (1994). Such instruments do not require strict monitoring of qualities since they give the right incentives to firms. However, the particular instrument to be used depends on the mode of competition prevailing in the industry. Indeed, in oligopolistic industries, the nature of the optimal policy depends on the mode of competition (see Brander and Spencer (1983)). In many instances, this information might be hard to identify accurately. The government must then cope with the risk of using the wrong instrument.

All in all, the two instruments most often studied, and used, in the field of quality regulation display some drawbacks which might call for investigating alternative instruments. The present paper aims at studying sales restrictions as an alternative instrument aimed at regulating quality provision. Its most appealing feature is that it overcomes the weaknesses of the two preceding ones. In opposition with MQS, sales restrictions provide firms with direct incentives to upgrade quality while preserving firms' margins. Moreover, contrary to tax/subsidy schemes and MQS, the impact of sales restrictions on optimal quality choices is largely invariant to the mode of competition. The key intuition underlying our analysis is the following: a sales restriction relaxes competition, whatever the mode of competition; therefore it increases the marginal profitability of quality upgrades and restores for all firms the incentive to select higher qualities. This is especially true for the low quality producer. Of course, a smoother competition might be detrimental to industry welfare but we show that in equilibrium, the positive impact of quality upgrades on welfare more than compensates the negative impact of reduced competition.

Clearly enough, sales restriction is not pervasive as a tool aimed at regulating quality provision in actual market. In this respect our contribution can be viewed as a purely speculative exercise. It is our contention that, given the limitations of MQS and taxes, there is room for purely theoretical exercises. Moreover, it should be stressed that there are many industries in which various forms of quantity regulations actually apply. For instance, supermarkets in most European countries face restrictions both on sizes and opening hours which actually limit their ability to preempt the market. In the Italian electricity market, generation companies are prohibited to install too large production capacities (their capacity is actually limited to half of the national market). A similar disposition can be found in the European regulation on the award of public service contracts passenger transport. According to this regulation, a public authority may indeed decide not to award contracts to an operator should this operator cover more than a quarter of the relevant market. The basic motivation for such regulations consists of guaranteeing through quantitative restrictions that there is enough room for competitors to survive in the market. We show that in concentrated industries this also relaxes competition and therefore promotes quality provision by all firms.

In order to capture formally the intuition underlying our proposal, we develop a very simple model. Suppose a government wishes to offer protection to an entrant facing com-

petition by an incumbent firm. We assume that the incumbent is already committed to its quality when the government implements a sales restriction. This allows us to concentrate on the quality selection of the entrant only. In order to analyze the role of the competition mode, we compare the optimal quality selection of the entrant under Cournot and Bertrand competition at the market stage, as a function of the “sales restriction” level. We show that for most values of this quota, the optimal quality selection of the entrant is the same under Cournot and Bertrand competition, and is greater than the optimal quality selected in the unconstrained game. Most often, the optimal quality level is negatively related to the tightness of the sales restriction. However, as compared to Cournot, the range where the quota is effective is larger under Bertrand competition.²

We present the model in section 2. Section 3 deals with Cournot competition whereas section 4 studies the Bertrand case. After comparing the two models in section 5, we conclude in section 6.

2 The Model

Consumers’ preferences are derived from the model of Mussa and Rosen (1978). Consumer j exhibits a taste for quality θ_j and derives an indirect utility $\theta_j s - p$ when consuming a product of quality s , bought at price p . Not consuming yields a utility of 0. Consumers’ types are uniformly distributed in the $[0, 1]$ interval. The density is 1, and is taken as a measure of the market size. In order to produce a quality level s , a firm has to incur a sunk cost $c(s) = \frac{s^2}{8}$.³ Marginal cost is assumed to be zero for simplicity.

An *incumbent* (denoted by i) originally sells as a monopolist in the market a product of quality $s_i = 1$.⁴ We assume the producer is committed to this quality level.

The sequence of decisions is the following. The government chooses a sales restriction for the incumbent (a quota) from an interval $[\underline{q}, 1]$ with $\underline{q} > 0$.⁵ Given the quota, the *entrant* (denoted by e) firm selects its quality s_e before both firms compete in the last stage of the game. We consider in turn Cournot and Bertrand competition. Notice that given the specification of the sunk cost, we may restrict the analysis of entrant’s quality selection to $s_e \leq 1$. Since the incumbent is committed to its quality, it is immediate to check that, under

²Notice that the impact of sales restrictions on quality selection has been studied recently in a specific context. Herguera, Kujal and Petrakis (2000) study the impact of a trade quota imposed on a foreign firm on equilibrium quality selection. However, their analysis is confined to Cournot competition. We compare our results to theirs in the last section of the paper.

³The analysis can be carried out with a more general convex cost. See Boccard and Wauthy (2001).

⁴Note that $s_i = 1$ is indeed the optimal choice of a monopolist under the cost assumption $c(s) = \frac{s^2}{8}$. The case for endogenous quality selection where the incumbent might not have the highest quality is the object of a more involved paper Boccard and Wauthy (2001).

⁵The introduction of a lower bound for the quota is made to avoid technical problems in the derivation of equilibria in the pricing games. The importance of this assumption is discussed below.

our assumption about quality costs, entering the market with $s_e > 1$ cannot be a best reply for the entrant. In other words, quality leapfrogging is not an issue in our basic model.

3 Cournot Competition

We study the quality decision of the entrant in two situations; firstly when the ensuing quantity competition is free from governmental intervention (hereafter unconstrained) and secondly when the regulation authority imposes a binding sales restriction (hereafter quota) to the incumbent firm.

3.1 Unconstrained Competition

We first solve the last stage of the game, and then we go backward to study quality selection. The analysis of this Cournot game is a standard exercise.⁶ Given qualities and prices, demands are

$$x_i = 1 - \frac{p_i - p_e}{1 - s_e} \quad (1)$$

$$x_e = \frac{p_i s_e - p_e}{s_e(1 - s_e)} \quad (2)$$

thus the inverse demands characterizing Cournot competition are given by

$$p_i = 1 - x_i - x_e s_e \quad (3)$$

$$p_e = (1 - x_i - x_e) s_e \quad (4)$$

The best replies in quantities are immediately derived as $BR_i^c(x_e) \equiv \frac{1 - x_e s_e}{2}$ and $BR_e^c(x_i) \equiv \frac{1 - x_i}{2}$. The unconstrained Cournot equilibrium is thus

$$x_e^c(s_e) \equiv \frac{1}{4 - s_e} \quad \text{and} \quad x_i^c(s_e) \equiv \frac{2 - s_e}{4 - s_e}. \quad (5)$$

leading to equilibrium prices

$$p_i^c = \frac{2 - s_e}{4 - s_e} \quad \text{and} \quad p_e^c = \frac{s_e}{4 - s_e}. \quad (6)$$

Notice that x_e^c is increasing with s_e while x_i^c is decreasing. The entrant's profits at the Cournot equilibrium are given by

$$\pi_e^c(s_e) \equiv \frac{s_e}{(4 - s_e)^2} - \frac{s_e^2}{8}. \quad (7)$$

The first order condition for maximization of π_e^c is $0 = 16 - 60s_e + 48s_e^2 - 12s_e^3 + s_e^4$ and letting $z \equiv \frac{2 + (33 - \frac{1}{9}\sqrt{22353})^{1/3} + (33 + \frac{1}{9}\sqrt{22353})^{1/3}}{2}$, the only relevant real root is

$$s_e^c \equiv 3 - \sqrt{z} - \sqrt{3 - z + 3z^{-1/2}} \simeq 0.36261 \quad (8)$$

⁶We may refer the reader unfamiliar with these models to Herguera et al. (2000) for a formal derivation of the demand functions

This defines the optimal quality selection by the entrant when no sales restriction is imposed. Demand addressed to the incumbent at the unconstrained SPE is $D_i^c \equiv x_i^c(s_e^c) \simeq 0.4502^7$ and the profit of the entrant is $\pi_e^c(s_e^c)$.

3.2 Constrained Competition

The regulation authority now imposes a sales restriction at the level q on the sales of the incumbent firm. In the analysis to follow we shall distinguish between two notions of effectiveness for the sales restriction.

A sales restriction is

★ short-run effective if it alters the price equilibrium outcome, given qualities.

★ long-run effective if it also alters the quality selection.

Suppose the quota is binding at the Cournot stage (after quality has been set), so that we have $x_i = q$. Against this quantity, the entrant's best reply is then $x_e = BR_e^c(q) = \frac{1-q}{2}$. Conditional on these quantities forming the constrained Cournot equilibrium, the first period profit of the entrant is

$$\pi_e^{cq}(s_e) \equiv \frac{(1-q)^2 s_e}{4} - \frac{s_e^2}{8}, \quad (9)$$

and is maximum for $(1-q)^2$ which is the optimal "quota-constrained" quality. We observe that this optimal quality is decreasing in the sales restriction i.e., the tougher the protection granted to the entrant, the greater the quality it can elicit.

Given the choice of quality s_e by the entrant, the quota is *short-run effective* if the incumbent unrestricted sales $x_i^c(s_e)$ are greater than q so that a necessary condition for this to happen is $q < 0.5$ (since $x_i^c \leq 0.5$). A quota below half of the market size is *short-run effective* if the subsequent quality choice satisfies $x_i^c(s_e) > q \Leftrightarrow s_e < 2\frac{1-2q}{1-q}$. This condition partitions the quality-quota space in two regions as shown on Figure 1 where the frontier is drawn in bold face. Notice that, given $q < \frac{1}{2}$, there always exists a value for quality below which the quota is short-run effective. Exploiting this property we may state a preliminary result.

Lemma 1 *Under Cournot competition, a quota set above the free-trade level D_i^c fails to be short-run effective whenever the quality is higher than s_e^c .*

Figure 1: Cournot competition

In order to identify the parameter space where the sales restriction is long-run effective, it remains to determine when it is optimal for the entrant to force a constrained Cournot

⁷In the remaining of the paper, we shall often appeal to approximate values for equilibrium outcomes. Note however that these approximations are corresponding to exact analytical values that we have chosen not to report systematically because of their length.

competition i.e., to take advantage of the sales restriction. The above figure shows indeed that it is always possible. However it does not mean that it is a wise strategy to follow since it often leads to quality downgrade and possibly lower profits.

The optimal quality choice in the non binding regime is the frontier for $q < D_i^c \simeq 0.450$ and s_e^c for larger q 's. Likewise the optimal quality choice in the binding regime is $(1 - q)^2$ for $q < \hat{q}$ and the frontier for larger q 's. The intersection of the frontier $2\frac{1-2q}{1-q}$ and the best reply $(1 - q)^2$ determines a threshold \hat{q} which solves $x_i^c((1 - q)^2) = q$; more precisely,

$$\hat{q} \equiv 1 - 1\sqrt{32} \cos \left(13 \arctan \left(\sqrt{1119} \right) \right) + 2 \sin \left(13 \arctan \left(\sqrt{1119} \right) \right) \simeq 0.461$$

It is easy to see on Figure 1 that the overall optimal choice is thus $(1 - q)^2$ if $q < D_i^c$ and s_e^c if $q > \hat{q}$. For $q \in [D_i^c; \hat{q}]$ we have two candidate equilibria thus we need to compare their respective profits $\pi_e^c(s_e^c)$ and $\pi_e^{cq}((1 - q)^2) = \frac{1}{8}(1 - q)^4$ to identify the overall optimal quality choice. Solving for q , we obtain as the unique relevant real root

$$q^c \equiv 1 - (8\pi_e^c(s_e^c))^{1/4} \simeq 0.4557 \in]D_i^c; \hat{q}[$$

Therefore q^c defines the critical value below which the sales restriction is long-run effective when committed to before the entrant's quality is selected. The next proposition characterizes the impact of a sales restriction under Cournot competition.

Proposition 1 *The equilibrium of the Cournot competition game depends on the sales restriction level q :*

- If $q \geq q^c$, the sales restriction is not effective, the unconstrained equilibrium obtains.
- If $q \leq q^c$, the sales restriction is long-run effective, the entrant's quality is $(1 - q)^2$.

Corollary 1 *When the sales restriction q becomes long-run effective, the entrant's quality falls from $s_e^c \simeq 0.363$ to $(1 - q^c)^2 \simeq 0.296$ as seen on Figure 1.*

Since government intervention tends to distort the competitive framework it is instructive to explore the effect of the sales restriction upon the total welfare generated in this market. Notice first that the first-best can be assimilated to a configuration where the incumbent sells a product of maximal quality at marginal cost (here zero) to obtain a consumer surplus of $\frac{1}{2}$ (see equation (17) in Appendix). Since there is a cost for quality of $\frac{1}{8}$, the maximal total welfare is 0.375. The following corollary is proven in Appendix.

Corollary 2 *welfare analysis for Cournot competition:*

- If $q > q^c$, given the equilibrium entrant quality s_e^c , industry welfare is $\bar{W}^c \simeq 0.2485$; it is independent of the sales restriction.
- If $q \leq q^c$, welfare increases with the quota and reaches the maximal value $\bar{W}^{cq} \simeq 0.2488$ for $q = q^c$.

The inequality $\bar{W}^{ca} > \bar{W}^c$ in a neighborhood of q^c is due to the fact that the quality downgrading boosts profits by increasing product differentiation more than the loss of consumer surplus which falls from 0.1599 to 0.1515.

As mentioned in the introduction, Herguera et al. (2000) consider a similar problem in the specific context of international trade. As compared to the free trade equilibrium, they show that the entrant is likely to select a higher quality in the presence of a binding quota (lesser than the entrant's sales under free trade). Yet if the quota is set just above the entrant's sales under free trade, the entrant selects a lower quality, as compared to free trade. This is precisely what we obtain. However, as we shall see in section 5, our analysis shows that the risk of a local quality downgrading is entirely specific to the Cournot framework.

4 Bertrand Competition

The analysis of Bertrand competition is slightly more complex than the Cournot one; it is divided in 4 parts. We first re-derive the unconstrained benchmark equilibrium, then tackle the implications of a sales restriction in a pricing game before solving it. Lastly, we turn to the choice of the optimal quality by the entrant.

4.1 Free trade

Under Bertrand competition, demands (1) and (2) enable to compute firms' best reply in the unconstrained pricing game as

$$\Phi_i(p_e) = \frac{1 - s_e + p_e}{2} \quad \text{and} \quad \Phi_e(p_i) = \frac{p_i s_e}{2}. \quad (10)$$

Straightforward computations yield the unique Nash equilibrium:

$$p_e^b = \frac{s_e(1 - s_e)}{4 - s_e} \quad \text{and} \quad p_i^b = \frac{2(1 - s_e)}{4 - s_e}. \quad (11)$$

Using these values, we compute the equilibrium sales: $x_e^b \equiv \frac{1}{4-s_e}$ and $x_i^b \equiv \frac{2}{4-s_e}$. The entrant's equilibrium profit is

$$\pi_e^b(s_e) \equiv \frac{s_e(1 - s_e)}{(4 - s_e)^2} - \frac{s_e^2}{8}. \quad (12)$$

Using equation (12), the optimal quality selection of the entrant, maximizing $\pi_e^b(s_e)$, is computed as

$$s_e^b \equiv 3 + \sqrt{y} - \sqrt{3 - y + 5y^{-1/2}} \simeq 0.1923$$

where $y \equiv 1 + 3^{-2/3}2^{-1} (5\sqrt{44889} - 423)^{1/3} - 34(5\sqrt{44889} - 423)^{-1/3}3^{-1/3}$.

The associated demand for the incumbent is $D_i^b \equiv x_i^b(s_e^b) \simeq 0.525$ while the profit of the entrant is $\pi_e^b(s_e^b)$. We observe that the tougher price competition forces the entrant who is a quality follower to differentiate more than under Cournot; this in turn enables the incumbent to increase sales with respect to the Cournot situation.

4.2 Effect of a sales restriction

Let us then consider the presence of a sales restriction. The first analysis of quota constrained pricing game with differentiated products has been developed in Krishna (1989) in a trade context.⁸ As shown by Krishna (1989), the first key difference between Cournot and Bertrand competition is that, under price competition, sales restrictions above the unconstrained equilibrium demand of the incumbent are (almost) always "short-run" effective. This is so because a sales restriction deeply alters the structure of the pricing game. Therefore, we first have to study in details the implication of the sales restriction on price competition itself before we turn to the analysis of quality selection.

The key point is the following: under price competition, consumers may be rationed by the incumbent if its demand exceeds the level of the sales restriction. Rationed consumers may then report their purchase on the entrant, whose effective sales increase. These rationing spillovers destroy the global concavity of the entrant's payoff and thereby make the existence of a pure strategy equilibrium problematic. Thus, the presence of the sales restriction induces Bertrand-Edgeworth competition at the market stage.⁹

A peculiarity of the present model is that if the entrant sells the low quality, any consumer that is rationed by the high quality incumbent prefers to report his purchase on the entrant's product rather than to refrain from buying. Accordingly, whenever $x_i(p_e, p_i) > q$, the number of consumers who report their purchase on firm e is given by $x_i(p_e, p_i) - q$. It is then direct to show that the residual demand addressed to the entrant is

$$x_e^q(p_e) \equiv 1 - q - \frac{p_e}{s_e}. \quad (13)$$

Using (13) we may define $p_e^q(p_i) \equiv p_i - (1 - q)(1 - s_e)$ by solving $x_i(p_e, p_i) > q$ for p_e to obtain $p_e > p_e^q(p_i)$. Whenever $p_e \leq p_e^q(p_i)$, the free trade analysis applies and firms' demand are given by (1) and (2). On the other hand, if $p_e \geq p_e^q$, the potential demand of the incumbent exceeds the legal limit q so that final sales are $x_e^q(p_e)$ and q respectively. Comparing the derivatives of the entrant's demand function using (1) and (13), we observe that the entrant's demand exhibits an outward kink at $p_e = p_e^q(p_i)$. This kink destroys the quasi-concavity of its profits. Thus, the existence of a pure strategy equilibrium is not guaranteed.

4.3 Price equilibrium

We now characterize the nature of equilibrium in the pricing game. The incumbent's best reply is denoted φ_i . Over the non-binding domain, the best reply is $\Phi_i(p_e)$ as defined by (10), if it belongs to this domain. Over the binding domain, the incumbent is better off selling

⁸We refer the interested reader to her paper for the full analysis of such cases. Regarding price equilibrium, the present analysis is a direct application of the methodology proposed by Krishna (1989).

⁹see Vives (2000) for a presentation of Bertrand-Edgeworth competition vs. Bertrand competition and why the former makes sense to adopt in the present setting.

q at the highest possible price, which is precisely the frontier price $p_i^q(p_e)$. The formal best reply is thus the minimum of $\Phi_i(p_e)$ and $p_i^q(p_e)$. Solving $\Phi_i(p_e) = p_i^q(p_e)$ for p_e we obtain $\tilde{p}_e = \max\{(1 - s_e)(2q - 1), 0\}$. Thus,

$$\varphi_i(p_e) = \begin{cases} \Phi_i(p_e) & \text{if } p_e < \tilde{p}_e \\ p_i^q(p_e) & \text{if } p_e \geq \tilde{p}_e \end{cases} \quad (14)$$

Notice that the best reply is kinked and continuous, reflecting the fact that the incumbent's profit is concave in own price.

We turn to the entrant whose profit when he benefits from spillovers is

$$p_e x_e^q(p_e) = p_e \left(1 - q - \frac{p_e}{s_e} \right)$$

The maximum of this expression is reached for $p_e^s \equiv \frac{1-q}{2}s_e$. Associated profits are equal to $\pi_e(q) = \frac{(1-q)^2}{4}s_e$. If the sales restriction is not binding, the best reply is $\Phi_e(p_i)$ as defined by (10), yielding a payoff $\pi_e(p_i) = \frac{p_i^2 s_e}{4(1-s_e)}$. Solving for equality among $\pi_e(q)$ and $\pi_e(p_i)$ in the variable p_i , we obtain the critical value $\tilde{p}_i \equiv (1-q)\sqrt{1-s_e}$ for which the entrant is indifferent between the two strategies. The best reply is therefore:

$$\varphi_e(p_i) = \begin{cases} p_e^s & \text{if } p_i \leq \tilde{p}_i \\ \Phi_e(p_i) & \text{if } p_i \geq \tilde{p}_i \end{cases} \quad (15)$$

Combining (14) and (15), we observe that there is only one candidate for a pure strategy equilibrium: the unconstrained equilibrium (p_i^b, p_e^b) as defined by (11).¹⁰ A necessary and sufficient condition for this candidate to be an equilibrium is that $p_i^b > \tilde{p}_i$. Direct computations show that this condition is satisfied if and only if $q > \bar{q}(s_e) \equiv 1 - \frac{2\sqrt{1-s_e}}{4-s_e}$. When this condition is not satisfied, there exist no pure strategy equilibrium. However, the continuity of payoffs ensures the existence of a mixed strategy equilibrium.

The natural candidate for a mixed strategy equilibrium is then the following one: the entrant randomizes between p_e^s and $\Phi_e(\tilde{p}_i)$ and chooses the weight to put on each pure strategy so as to ensure that \tilde{p}_i is indeed a best reply for the foreign firm against the mixture. We call this equilibrium the Krishna equilibrium.¹¹ The following lemma summarizes the key difference between quantity and price competition in the presence of quantitative constraints. This result is central to the analysis originally proposed in Krishna (1989):

Lemma 2 *As opposed to the Cournot case, a sales restriction above the unconstrained equilibrium level can be short-run effective in the Bertrand game whatever the quality level of the entrant.*

¹⁰It is indeed immediate to check that $\tilde{p}_i < p_i^q(p_e^s)$, which is sufficient to rule out any pure strategy equilibrium candidate in the sales restriction binding domain of prices. Therefore, the only remaining candidates must lie in the unconstrained region. There is only one such candidate: the unconstrained equilibrium.

¹¹We refer the interested reader to Krishna (1989), Theorem 2, for the detailed construction of this equilibrium.

Proof Direct computations show that $\bar{q}(s_e) > x_i^b(s_e) = \frac{2}{4-s_e}$ is satisfied for all s_e . Accordingly, whatever the entrant's quality level s_e , any sales restriction $q \in [x_i^b(s_e); \bar{q}(s_e)]$ is such that the price equilibrium is a mixed strategy equilibrium involving prices greater than those of the unconstrained equilibrium. In particular, at the unconstrained SPE quality level s_e^b we have $\bar{q}(s_e^b) = 0.528 > x_i^b(s_e^b) = 0.525$. ■

4.4 Quality selection

Let us assume for the moment that the Krishna equilibrium always exists when the unconstrained one does not and study the issue of quality selection. It is not necessary to compute the mixed strategy explicitly for our present purpose. Indeed, the key point here is to note that in this equilibrium, the entrant earns exactly $\pi_e^s = \frac{(1-q)^2}{4}s_e - \frac{s_e^2}{8}$ in the Krishna equilibrium.¹² Notice that this is exactly equivalent to the Cournot equilibrium payoffs under a binding sales restriction. Therefore, if the Krishna equilibrium is played at the price competition stage, the optimal quality is $(1-q)^2$, yielding the net payoff $\frac{(1-q)^4}{8}$.

In order to delineate the domain in which the sales restriction affects quality selection, consider the optimal quality selection in the unconstrained equilibrium: $s_e^b \simeq 0.192$. As noted above, we need $q > q^b(s_e^b) = 0.528$ for the unconstrained equilibrium to exist in the pricing game. On the other hand, it is always possible for the entrant to enforce the Krishna equilibrium by choosing a high enough quality. In order to identify the optimal strategy, we compare the corresponding payoffs. Direct computations indicate that $\pi_e^b(s_e^b) < \frac{(1-q)^4}{8}$ whenever

$$q < q^b \equiv 1 - \left(8\pi_e^b(s_e^b)\right)^{1/4} \simeq 0.530,$$

where q^b defines the critical value above which the sales restriction is neither long-run nor short-run effective under Bertrand competition. Our findings are summarized in the following proposition.

Proposition 2 *The equilibrium of the Bertrand competition depends on the sales restriction level:*

- If $q \geq q^b$, the sales restriction is not effective, the unconstrained equilibrium s_e^b obtains.
- If $q \leq q^b$, the sales restriction is long-run effective, entrant's quality is $(1-q)^2$.

Let us then briefly comment on the existence of the Krishna equilibrium. Indeed, this equilibrium does not always exist. For this equilibrium to exist, the incumbent's demand must satisfy the non-negativity constraint $x_i(\Phi_e(\tilde{p}_i), \tilde{p}_i) > 0$. Direct computations show that this is the case if only

$$q > 1 - 2\sqrt{1-s_e}. \tag{16}$$

¹²In a mixed strategy, the equilibrium payoff can be computed at any of the firm's atom. We do it for the entrant with p_e^s taking into account that the incumbent plays the pure strategy \tilde{p}_i .

In other words, there exists a lower bound on the sales restriction value below which the Krishna equilibrium fails to exist. If $s_e < 3/4$ then (16) is satisfied since its RHS is negative. For $s_e \geq 3/4$ we need to check condition (16) applied to the equilibrium quality $(1 - q)^2$. We obtain:

$$(1 - q)^2 < 1 - \frac{(1 - q)^2}{4} \Leftrightarrow q > 1 - \frac{2}{\sqrt{5}} \simeq 0.106$$

As a consequence, our previous analysis is fully compelling if we assume $\underline{q} \geq 0.106$. Hence our initial assumption of a lower bound on the admissible values for the sales restriction.¹³ Since competition is tougher under the Bertrand regime, differentiation is greater than under the Cournot regime ($s_e^b < s_e^c$). One interesting difference between the two regimes is the fact that a sales restriction (whenever effective) gives protection to the entrant who can therefore increase its quality.

Corollary 3 *When the sales restriction q becomes long-run effective, the entrant's quality rises from $s_e^b \simeq 0.192$ to $(1 - q^b)^2 \simeq 0.221$.*

As in the previous section, we can perform a welfare analysis. It is more involved than in the Cournot case because the equilibrium is in mixed strategies when the quota is binding and this makes the computation of profits and consumer surplus more lengthy. The algebra is deferred to the appendix; it yields an interesting result: setting an effective sales restriction strongly increase welfare by enabling the entrant to upgrade its quality.

Corollary 4 *welfare analysis for Bertrand competition:*

- If $q \geq q^b$, given the equilibrium entrant quality s_e^b , welfare is $W^b \simeq 0.275$; it is independent of the sales restriction.
- If $q \leq q^b$, welfare increases with the quota and reaches the maximal value $\bar{W}^{bq} \simeq 0.286$ for $q = q^b$.

5 Comparing Cournot and Bertrand

As shown by Krishna (1989), the impact of a sales restriction at the market stage depends on the mode of competition. This result is best observed in our framework by noting that a sales restriction set slightly above the unconstrained SPE sales of the incumbent $D_i^c \simeq 0.450$ is never short-run effective under Cournot while it is always short-run effective under Bertrand (provided the entrant's quality is optimally set). However, a sales restriction has also long-run implications. By comparing quality selection by the entrant in the two competition

¹³When the Krishna equilibrium does not exist, there exists fully mixed strategy equilibria involving a finite number of atoms. We have not been able to fully characterize them. However, Levitan and Shubik (1972) provide a characterization for the particular case where $s_e = 1$. For the relevant domain of sales restriction values, the payoff of the entrant at $s_e = 1$ dominates the unconstrained benchmark. Hence, in this domain, it must be the case that the equilibrium quality is larger than the unconstrained one.

frameworks, we may now assess the long-run effectiveness of sales restrictions with the help of Figure 2 below.

Figure 2: Comparing the two modes of competition

We obtain two results. Firstly $q^b \simeq 0.530 > q^c \simeq 0.456$ implies that the range for an effective sales restriction is larger under Bertrand than under Cournot. Secondly, the quality selection is invariant to the mode of competition for a large domain of sales restrictions ($q < q^c$). Our claim that imposing a “sales restrictions” on an incumbent firm can enhance the quality provision of entrants is best understood with the help of Figure 2: it is enough to set a quota q^o to guarantee that, whatever the mode of competition, the lowest quality is s_e^c . The regulator can increase quality simply by making the sales restriction tighter.

As compared to their respective unconstrained values, the optimal quality cannot decrease because of the sales restriction under Bertrand competition while there exists a domain of sales restriction values, above the Free Trade benchmark for which the sales restriction induces quality downgrading under Cournot. The intuition underlying our result is easy to summarize. The presence of the sales restriction softens competition, especially from the point of view of a low quality firm. Under Cournot competition, this increases the marginal values of quality upgrading. Under Bertrand competition, the effect is even more striking. Sales restrictions relax price competition so drastically that they remove any strategic incentives to select a low quality level. There is indeed no need to relax price competition further.

The welfare analysis has followed the utilitarian approach where 1€ of profits counts like 1€ of consumer surplus counts. If the good or service delivered in this market is believed to produce positive externalities, as it would be the case for education or health, then the consumer surplus should receive a greater weight; this would only reinforce our argument that the sales restriction is a helpful policy instrument to achieve the optimal quality without jeopardizing the survival of firms. Indeed, whatever the mode of competition, the quota smoothes out rivalry by guaranteeing minimal market shares.

6 Conclusion

The present article aimed at providing a simplified example in which the role of “sales restrictions” as a mean to regulate quality provision could be exposed. Compared to MQS or tax instruments, sales restrictions exhibit two advantages: first they are effective whatever the mode of competition and, second, they do not depress firms’ margins.

This mechanism might seem rather unusual at first sight and is clearly at odds with most regulation actually in force regarding quality provision. At a theoretical level, our analysis shows however that it is very effective because it alters competition in a way that provides firms with the desired incentives regarding quality choice. Regarding the actual applicability

of this instrument, it is first useful to recall that regulation authorities often use quantity regulation instruments (although for different purposes). For instance, trade quotas have a long tradition which in particular suggests that monitoring sales is often a feasible option. Glaeser and Shleifer (2001) argue that such instruments might be superior because they are less costly to enforce. Whether monitoring sales rather than quality levels is easier should be assessed on an empirical basis. It seems however that controlling sales levels may require less sophisticated inquiries than quality levels.

Quantitative restrictions have also been enforced in recently deregulated industries. As reported for instance by McDaniel and Neuhoff (2002), the British monopolies and merger commission forced British Gas to contract no more than 90% of new gas fields in order to favor competition. Clearly, we do not argue that energy or basic services industries should be regulated by drastically limiting the size of operators. Instead, we should stress that the level of sales restriction which is necessary to enforce quality upgrading is actually quite loose, so that at the price equilibrium, the restriction appears most often to be non-binding. By limiting the capacity of the incumbent operator, the regulator actually offers protection to the entrant, which in turn find it profitable to sink money into quality upgrades.

Therefore, it is our belief that such a mechanism is more than a theoretical curiozum and worth being investigated further. Indeed, our results have been obtained in a highly stylized framework. Several generalizations of our example can be considered. Shouldn't the two firms be constrained? Would a firm, or two, voluntarily choose to constrain its sales? A common feature of these generalizations is that they require a more general analysis of sales-constrained pricing games with differentiated products. The characterization of price equilibria in such settings is on our research agenda.

Assuming that only two firms are active is also a limitation. This is especially important if we want to assess the dynamic impact of regulation on entry. For instance, it has been argued that a drawback of MQS was that it reinforces entry barriers (Maxwell (1998)). In this respect, observe that sales restrictions may lead to lower market covering (but since average quality increases, the consumer surplus also increases). Regarding entry, this might be good news because the largest the non covered segment, the larger the incentive for a third firm to enter this market. Entry deterrence might therefore become more costly.

Fairly enough, assuming that the incumbent does not alter its quality selection as a response to the sales restriction considerably eases the analysis. However, it is our belief that these results capture some basic implications of "sales restrictions" on quality selection in more general settings. In Bocard and Wauthy (2001), we consider a trade game where the domestic and the foreign firms are free to choose any quality level, and thus induce any quality ranking between the domestic and the foreign product after the sales restriction is implemented. We reach qualitatively similar conclusions: incentives to quality selection under Bertrand competition are in line with those prevailing under Cournot whenever the

sales restriction is effective. The degree of product differentiation decreases when the sales restriction becomes tighter. When it becomes long-run effective, a sales restriction induces a marked quality upgrading under Bertrand. In other words the impact of a quantitative restriction on quality selection seems to be qualitatively robust.

7 Appendix

In this appendix we derive the welfare analysis.

7.1 welfare analysis for Cournot competition

The consumer surplus in a vertical differentiation model is

$$CS = \int_{1-x_i}^1 (x - p_i) dx + \int_{1-x_i-x_e}^{1-x_i} (xs_e - p_e) dx$$

and is equal to

$$\int_0^1 x dx = \frac{1}{2} \quad (17)$$

if sold at marginal cost (here zero) to all consumers by the top quality firm.

7.1.1 Non binding restriction

When $q \geq q^c$, quantities are $x_e^c(s_e) = \frac{1}{4-s_e}$, $x_i^c(s_e) = \frac{2-s_e}{4-s_e}$, prices are $p_i^c = \frac{2-s_e}{4-s_e}$ and $p_e^c = \frac{s_e}{4-s_e}$, hence

$$\begin{aligned} CS^c(s) &= \int_{1-\frac{2-s}{4-s}}^1 \left(x - \frac{2-s}{4-s} \right) dx + \int_{1-\frac{2-s}{4-s}-\frac{1}{4-s}}^{1-\frac{2-s}{4-s}} s \left(x - \frac{1}{4-s} \right) dx \\ &= \frac{4+s(1-s)}{2(4-s)^2}. \end{aligned}$$

Since $\pi_i^c(s_e) = \left(\frac{2-s_e}{4-s_e} \right)^2 - 1/8$ and $\pi_e^c(s_e) \equiv \frac{s_e}{(4-s_e)^2} - \frac{s_e^2}{8}$, total welfare is

$$\begin{aligned} W^c(s) &= CS^c(s) + \pi_e^c(s) + \pi_i^c(s) \\ &= \frac{48 - 20s - 12s^2 + 8s^3 - s^4}{8(s-4)^2} - \frac{1}{8}. \end{aligned}$$

Given the entrant's optimal quality selection s_e^c , welfare is $\bar{W}^c \simeq 0.24851$.

7.1.2 Binding restriction

When $q \leq q^c$, quantities are $x_i = q$ and $x_e = \frac{1-q}{2}$, prices are $p_i^{cq} = (1-q)(1-\frac{s_e}{2})$ and $p_e^{cq} = \frac{1-q}{2}s_e$, thus

$$\begin{aligned} CS^{cq}(s) &= \int_{1-q}^1 (x - (1-q)(1-s_2)) dx + \int_{\frac{1-q}{2}}^{1-q} s(x - 1 - q_2) dx \\ &= \frac{s}{8}(2q - 3q^2 + 1) + \frac{1}{2}q^2 \end{aligned}$$

As $\pi_e^{cq}(s_e) = \frac{(1-q)^2 s_e}{4} - \frac{s_e^2}{8}$ and $\pi_i^{cq}(s_e) = q(1-q) \left(1 - \frac{s_e}{2}\right) - 1/8$, total welfare is

$$\begin{aligned} W^{cq}(s) &= CS^{cq}(s) + \pi_e^c(s) + \pi_i^c(s) \\ &= \frac{3(1-q)^2 s - s^2 + 3 - 4(1-q)^2}{8} \end{aligned}$$

and since the entrant optimal quality is $s_e^{cq} = (1-q)^2$, welfare, as a function of the sales restriction q , is

$$W^{cq} = W^{cq}(s_e^{cq}) = \frac{1}{8} + q^2 - q^3 + \frac{1}{4}q^4;$$

it is increasing over $[0; q^c]$ and reaches the maximal value $\bar{W}^{cq} \simeq 0.24881$ for $q = q^c \simeq 0.4557$.

7.2 welfare analysis for Bertrand competition

7.2.1 Non binding restriction

When $q \geq q^b$, equilibrium prices are $p_e^b = \frac{s_e(1-s_e)}{4-s_e}$ and $p_i^b = \frac{2(1-s_e)}{4-s_e}$ and sales are $x_e^b = \frac{1}{4-s_e}$ and $x_i^b = \frac{2}{4-s_e}$, thus the consumer surplus is

$$CS^b(s) = \int_{1-\frac{2}{4-s}}^1 (x - 2(1-s)4-s) dx + \int_{1-\frac{2}{4-s}-\frac{1}{4-s}}^{1-\frac{2}{4-s}} s(x - (1-s)4-s) dx = \frac{5s+4}{2(s-4)^2}.$$

Since profits are $\pi_e^b(s_e) = \frac{s_e(1-s_e)}{(4-s_e)^2} - \frac{s_e^2}{8}$ and $\pi_i^b(s_e) = \frac{4(1-s_e)}{(4-s_e)^2} - \frac{1}{8}$, total welfare is

$$\begin{aligned} W^b(s) &= CS^b(s) + \pi_e^b(s) + \pi_i^b(s) \\ &= \frac{4s + 32 - 25s^2 + 8s^3 - s^4}{8(4-s)^2} \end{aligned}$$

thus $W^b = W^b(s_e^b) = 0.27503 > W^c$.

7.2.2 Binding restriction

When $q \leq q^b$, the analysis is more involved because to compute the profit of the incumbent and the consumer surplus we must derive the mixed strategy equilibrium precisely. The entrant randomizes between $p_e^s = \frac{1-q}{2}s_e$ (probability α) and $\Phi_e(\tilde{p}_i) = \frac{\tilde{p}_i s_e}{2}$ (probability $1-\alpha$) while choosing the weight α to ensure that \tilde{p}_i is indeed a best reply for the foreign firm against the mixture.

When facing p_e^s the demand addressed to firm i is $x_i = q$ while it is $x_i = 1 - \frac{p_i - \Phi_e(\tilde{p}_i)}{1-s_e}$ when facing $\Phi_e(\tilde{p}_i)$. The expected profit is thus

$$\begin{aligned} \pi_i^b(s_e, p_i) &= p_i [\alpha x_i(p_i, p_e^s) + (1-\alpha)x_i(p_i, \Phi_e(\tilde{p}_i))] - \frac{s_e^2}{8} \\ &= p_i \left[\alpha q + (1-\alpha) \left(1 - \frac{p_i - \Phi_e(\tilde{p}_i)}{1-s_e} \right) \right] - \frac{s_e^2}{8} \end{aligned}$$

and is maximum when $\alpha q + (1-\alpha) \left(1 - \frac{2p_i - \Phi_e(\tilde{p}_i)}{1-s_e} \right) = 0$ i.e., for

$$p_i = \frac{\Phi_e(\tilde{p}_i)}{2} + \frac{1-s_e}{2} \left(\frac{\alpha q}{1-\alpha} + 1 \right) = \frac{\tilde{p}_i s_e}{4} + \frac{1-s_e}{2} \left(\frac{\alpha q}{1-\alpha} + 1 \right).$$

Now, in equilibrium, α is such that this best reply is exactly \tilde{p}_i hence

$$\alpha = \frac{\frac{\tilde{p}_i}{2} \left(\frac{4-s}{1-s} \right) - 1}{\frac{\tilde{p}_i}{2} \left(\frac{4-s}{1-s} \right) - 1 + q}.$$

The equilibrium profit is then

$$\begin{aligned} \pi_i^b(s_e) &= \frac{2q\tilde{p}_i^2}{\tilde{p}_i(4-s_e) - 2(1-q)(1-s_e)} - \frac{s_e^2}{8} \\ &= \frac{2q(1-q)\sqrt{1-s_e}}{4-s_e - 2\sqrt{1-s_e}} - \frac{s_e^2}{8} \end{aligned}$$

using the precise expression of $\tilde{p}_i = (1-q)\sqrt{1-s_e}$.

We can also compute the expected consumer surplus

$$CS^b(s_e) = \alpha CS(s_e, \tilde{p}_i, p_e^s) + (1-\alpha)CS(s_e, \tilde{p}_i, \Phi_e(\tilde{p}_i))$$

where $CS(s_e, p_i, p_e) = \int_{1-x_i}^1 (x-p_i)dx + \int_{1-x_i-x_e}^{1-x_i} (xs_e - p_e)dx$. Hence,

$$\begin{aligned} CS(s_e, \tilde{p}_i, p_e^s) &= \int_{1-q}^1 (x-\tilde{p}_i)dx + \int_{\frac{p_e^s}{s_e}}^{1-q} (xs_e - p_e^s)dx \\ &= \frac{q}{2}(2-q-2\tilde{p}_i) + \frac{s_e}{8}(1-q)^2 \end{aligned}$$

and

$$\begin{aligned} CS(s_e, \tilde{p}_i, \Phi_e(\tilde{p}_i)) &= \int_{\frac{p_i-p_e}{1-s_e}}^1 (x-\tilde{p}_i)dx + \int_{\frac{p_i-p_e}{1-s_e} - \frac{p_i s_e - p_e}{s_e(1-s_e)}}^{\frac{p_i-p_e}{1-s_e}} (xs_e - \tilde{p}_i s_e)dx \\ &= \int_{\frac{p_i}{2} \frac{2-s_e}{1-s_e}}^1 (x-\tilde{p}_i)dx + s_e \int_{\frac{p_i}{2}}^{\frac{p_i}{2} \frac{2-s_e}{1-s_e}} (x-\tilde{p}_i s_e)dx \\ &= \frac{1}{2} - \tilde{p}_i - \frac{(2s_e^2 + s_e - 4)\tilde{p}_i^2}{8(1-s_e)} \end{aligned}$$

so that, using the precise expression of $\tilde{p}_i = (1-q)\sqrt{1-s_e}$, we obtain

$$CS^b(s) = \frac{1}{8}(4-s)(q(2-q)(4-s)+s)\sqrt{1-s} + 2(1-s)(-2qs^2 + 2q^2s^2 + 4qs - 3q^2s - s + 12q^2 - 16q)\sqrt{1-s}$$

Finally total welfare $W^b(s_e) = CS^b(s_e) + \pi_i^b(s_e) + \pi_e^b(s_e)$ can be computed for the optimal quality choice $(1-q)^2$ to yield an increasing function of the quota, reaching approximatively 0.286 at q^b .

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