

Repeated Games: Homework 1.

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December 2009

1.a. (Tirole's IO book, exercise 6.4. (page 250)). Consider infinitely repeated Bertrand competition between n firms who all have a constant marginal cost $c > 0$. The market demand at time t , given prices (p_1^t, \dots, p_n^t) , is equal to

$$D(\min_{p_i^t} \{p_1^t, \dots, p_n^t\}) \times \mu^t$$

where, for example, if $t = 5 : \mu^t = \mu^5$. We suppose that $\mu\delta < 1$. When two or more firms price the lowest price, then they share the market. Derive the set of discount factors such that full collusion (monopoly pricing each period) is sustainable as a SPE of the repeated game. What would this model predict about the relative ease to collude in booms or recessions?

1.b. Consider the standard version of the Bertrand model in part 1.a. That is, we set $\mu = 1$. Determine the conditions on the discount factor needed to support prices which are higher than the monopoly price as a SPE of the repeated game. Show that, in this case, higher and higher prices require higher levels of patience. Does the same hold for prices between marginal cost and the monopoly price?

2. Take the standard infinitely repeated game, as defined in class, such that $A = \prod_{i=1}^n A_i$ is finite. Show that if δ is close enough to zero then every SPE requires the play of a static NE in every period.