Delegation and Information Revelation*

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Abstract

This paper addresses the issue of delegation in a partial contracting set-up, where only the control over actions is contractible. We consider an organization that must take, in sequence, two decisions, whose payoffs depend on a random parameter. The realization of this parameter is known only to the agent. If the principal gives up the control over the first decision to the better informed agent, his choice of the project may signal his private information. This revelation of information, induced by delegation, is valuable to the principal if she retains control over the second decision. Hence, this paper provides a new rationale for delegation: A transfer of control to the better informed party can be used by the principal to elicit the agent’s private information. We show that this partial delegation contract can dominate a centralized structure where the principal takes all the decisions after having been advised by the agent.

Keywords: Delegation, Asymmetric information, Signaling

JEL codes: D23, D82, L22
1 Introduction

This paper studies the problem of information transmission within an organization. We consider a repeated relationship between a better informed agent and a principal (for convenience, in the sequel, we will refer to the principal as "she" and to the agent as "he"). The organization must choose, in sequence, two projects (decisions/actions) whose payoffs are affected by the realization of a unique random parameter. Initially, there is asymmetric information between the principal and the agent: The agent knows the realization of the parameter, the principal does not. Moreover, they have diverging interests, i.e., they do not agree on the optimal projects.

In a complete contract set-up, the principal designs a revelation contract that specifies the actions that should be taken by the agent and the corresponding payment as a function of them. In this paper, we adopt the incomplete contract view of organizations and assume the projects, and the underlying economic environment (the "state of the world"), cannot be contracted for, neither ex-ante nor ex-post. In the terminology of incomplete contract theory, we consider the case of observable, but non-verifiable, decisions. However, we assume that control over projects can be contracted for. In our setting, the only feasible contracts are the ones stipulating who is in charge of each project. At the beginning of the game, the principal allocates the "right" to undertake project(s) either to herself or to the agent.

In the absence of performance-based payments, the "organization" of the decision-making process is crucial for the global performance of an organization. The informational asymmetry can be overcome either directly (with a message sent by the agent to the principal who then decides) or indirectly (the actions of the agent reveal his private information to the principal). In our set-up, these two alternatives are associated with two different organization structures: "centralization" and "partial delegation". This paper compares these two organizational structures. It shows that they are associated with different transfers of information. Under partial delegation, the principal can learn the true state by observing the agent's action while, under centralization, the principal could remain imperfectly informed. Information transfer is valuable for the principal in a context of repeated relation. Hence, the principal could prefer partial delegation to an agent with different preferences to centralization because it is a mean to learn the agent's private information.

In the case of centralization, the principal chooses both projects and the agent may transmit a message containing information about the actual state of the world (for instance, the agent can be asked to advise the principal). Given that the principal cannot build a revelation contract, the communication of the agent is strategic: He does not necessarily want to communicate the true information. Instead, he will communicate the information that best fits his interests, when used by the principal to select the projects. Communication under centralization is a cheap-talk game and, as shown by Crawford and Sobel (1982), this kind of communication is noisy.

In the case of partial delegation, the principal delegates the choice of some projects to the agent. By observing the agent's choice in the first period, she may acquire (part of) the agent's superior information. The acquisition of information is valuable for her only if she can use it to decide in the second period and, therefore, delegation must be only partial: The principal gives up control over the first period project, but keeps control over the second period one. The delegation mechanism is costly for her. When she gives up the right to decide, the agent does

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1 Baron and Myerson (1982).
3 This kind of contract is called "partial contracting" by Aghion, Dewatripont and Rey (2002).
4 Partial delegation is then a signaling game (Spence, 1973).
not implement her preferred decision. Diverging interests between the parties create losses due to her lack of control when she delegates. The principal’s lack of commitment implies that the agent cannot be punished for it.

Our main result is that (partial) delegation is a mechanism to induce full revelation by the better informed agent. We establish this result using an appropriate equilibrium refinement, i.e., Cho and Kreps (1987) intuitive criterion. Hence, when the principal cannot elicit the agent’s information through communication, she can do it by giving up control over the first project. Contrary to the complete contract framework, where the principal can control the rents she leaves to the agent, there is loss of control in our mechanism as soon as the principal gives power to the agent.\(^5\)

In our model, the principal has two conflicting objectives: She needs to elicit the agent’s private information to select the best projects and she would like to retain control over the choice of the projects to avoid loss of control. A partial transfer of control to the agent can partially meet both objectives: She keeps some control and she also becomes informed. However, the delegation mechanism is costly for her: The agent must receive decision-making power to reveal his information and, consequently, the principal suffers losses due to her lack of control. Hence, there is a trade-off between giving up power and acquiring information. Still, partial delegation can be the optimal organizational structure. This is true when information is not revealed in the message game and the losses due to the lack of control are small relative to the benefits of acquiring information. Hence, depending on the value of the agent’s private information and the divergence of interests between the two parties, the principal either prefers the centralized decision-making process or the one where the agent partially receives the control.

The results of this paper are built on the assumptions that the decisions/projects cannot be described in a comprehensive complete contract. They potentially apply to various forms of organizations/relationships. The literature on strategic communication provides examples of relations between parties with different information and in which projects cannot be contracted for. Ottaviani (2000) studies the relation between a better informed financial advisor and an investor. Dessein (2002) and Morris (2001) illustrate their models with examples from political science. In particular, one of the example in Morris (2001) concerns a repeated relation between a social scientist (advisor) and a politician (decision-maker). The social scientist has a better knowledge of the “state of the world” and he is asked to advise repeatedly the politician. We consider an alternative decision-making process in which the scientist is asked by the politician to fully design a project in his field of expertise. The project is then implemented by the politician. In this case, the scientist has the real authority over the decision though he has not the formal authority.\(^6\) By delegating the project design to the social scientist, the politician improves her knowledge of the field and she can use it to take subsequent decisions. Aghion and Tirole (1997) also study relationships where the allocation of power is the only tool available in a context of asymmetric information. Clearly, all these examples could fit our model. To make our point clear, the following stories could be used to illustrate the main ideas behind our model.

Manager-Shareholders relationship. Managers are supposed to act in the interests of the shareholders. However, since Berle and Means (1932), it is usually taken for granted that ownership (shareholders) and control (managers) have different objectives. In practice, shareholders often transfer the effective control over the firm to the managers and rubber stamp most of the

\(^5\)With partial delegation, the principal learns the agent’s private information by giving him control in the first period. This is in sharp contrast with the complete contract literature result that information revelation could be delayed (the so-called ratchet effect), see Freixas et al. (1985), Laffont and Tirole (1988).

\(^6\)Aghion and Tirole (1997).
managers main decisions. A reason for that is the managers superior information about the firm and its environment. Contracts usually fail to reconcile the interests of the two parties. Thereby, for maximizing the firm’s value, shareholders have to decide which decisions they leave to the manager and which decisions they keep in their hand. Whether or not shareholders rubber stamp the managers decisions depends upon the information they have, and presumably the available information depends on the past actions of the manager. For instance, if the manager decides to acquire a firm in a new business segment, shareholders may allow the manager to diversify the firm as they do not have the necessary information. However, if later on the manager suggests additional acquisitions in the same field, shareholders may have more information to assess the benefits of such a merger and may exercise more control on this second merger decision. This shared control situation with information transfer is precisely what we describe in this paper.

Advisors. Many organizations (for instance, firms, governments or international organizations) delegate research to experts such as consulting firms or universities while they have the resources to produce it. We can explain such a delegation by the fact that even if the consulting firms and/or universities have a different objective from those of the government (for example valuable academic research vs. support to decisions), they have a better knowledge of the state of research in some particular field. Part of this expertise is transmitted through the research output and the decision-makers can use it as a basis for their subsequent research that can be oriented more precisely toward their own aims.

In the literature, the choice between delegation and centralization is often a simple trade-off between losses due to the lack of control associated with delegation and informational benefits, when the delegate is better informed than his supervisor. The benefits of a delegated structure could be better communication (Melumad, Mookherjee and Reichelstein (1992)), better ability to prevent collusion (Laffont and Martimort (1998) and Felli (1996)), an informed decision-maker (Legros (1993), Dessein (2002), Ottaviani (2000)) or increased incentives provided to the agent (Aghion and Tirole (1997)). Most of these papers compare delegation and centralization, but they do not consider delegation as a tool to transmit information from the better informed agent to his supervisor. Dessein (2002) and Ottaviani (2000) consider a one period game where the principal can either delegate the project choice to a better informed agent with biased preferences or take the decision herself and use the agent’s advice to improve her knowledge of his private information. As in our framework, communication under centralization is not fully informative. The decision to delegate, or not to, depends on the trade-off between loss of control and loss of information. Losses due to the lack of control occurs when an informed agent receives control, losses due to the lack of information occurs when the informed agent advises the principal, who then chooses the project. Our paper introduces a second period in this model, so that delegation becomes an instrument to elicit the agents hidden information. Legros (1993) integrates this dimension. In his two period model, the principal delegates the first project choice to a better informed agent chosen at random in a given set of agents. In the second period, the principal can either re-select the same agent and let him choose the second project or she can hire a new agent, chosen at random, to implement the second decision. In this model, the first period decision does not perfectly signal the agents private information (i.e., his type) to the principal. This result differs from ours because in Legros model the agent trades off the immediate benefit of implementing his preferred policy (revealing his private information) with the probability of being re-selected, which increases when he can convince the principal that his preferences are close to her. Hence, the first period decision is a not a perfect signal of the agents type.

The paper is organized as follows: The model is presented in section 2. The outcomes under
delegation and centralization are described in sections 3 and 4. Section 5 compares the two organizational structures. The main conclusions are in section 6.

2 The Model

Consider an organization composed of one principal and one agent. The organization chooses two projects \( d_1 \) and \( d_2 \) at period \( t = 1 \) and \( t = 2 \), respectively. The selection of a project affects the welfare of the principal and the agent. Their utility levels are also affected by a common environmental parameter, \( \theta \), which realizes before any choice is made. The payoff of the project depends on the economic environment.

Environmental parameter The payoff of the project depends on the economic environment. For simplicity, we assume that a state of the world is fully described by a parameter, \( \theta \in \Theta = \{ \theta_L, \theta_H \} \), with \( \theta_H > \theta_L \). The principal only knows the realization \( \theta \). The principal only knows \( \Theta \) and the (common knowledge) distribution \( F(\theta) \). Let \( v_L \) be the probability of \( \theta = \theta_L \) and let \( v_H = 1 - v_L \). Moreover, let \( \Delta \theta = (\theta_H - \theta_L) \). We briefly discuss in the conclusion the extension to \( N > 2 \) states of the world.

Projects At each period \( t \), the decision, \( d_t \), is the choice of a project to be implemented by the organization, with \( d_t \in (0, +\infty) \).

Allocation of the rights to choose the projects We assume that projects can be observed by both parties, but cannot be contracted for (using the terminology of Tirole (1999), they are observable, but not verifiable). As in Dessein (2002), the realization of \( \theta \) cannot be contracted upon. Hence, the only variable agents can contract upon is the right to choose the projects \( d_1 \) and \( d_2 \). These control rights are allocated by the principal either to herself (centralization) or to the agent (delegation).

These contractual restrictions are consistent with the incomplete contract view of organizations: To give authority to a subordinate agent means to give him the right to make a decision within an allowed set (see, Simon (1958), Grossman and Hart (1986), Hart and Moore (1988), Aghion and Tirole (1997)).

There are four possible allocations of decisions right: The principal keeps control over both projects (centralization); she delegates both choices; the control rights are split between principal and agent (\( d_1 \) or \( d_2 \) is delegated). We call partial delegation the case in which the better informed agent receives control over \( d_1 \) (but not over \( d_2 \)). In the model, we concentrate mainly on centralization and partial delegation. Complete delegation and partial delegation of \( d_2 \) will be discussed at the end of the paper.

Timing of the events

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7 This is a simplification. Alternatively, we could assume that the state of the world changes over periods and that there is some correlation between the states of the world in the two periods. The results of the paper would remain qualitatively the same. The important assumption is that the observation of the first decision (under delegation) improves the information about the state of the world in the second period.

8 This is a common assumption in dynamic models of incentive contracts. See, e.g., Laffont and Tirole (1988).

9 The fact that the principal initially possesses decision rights over both projects can be justified by her ownership of the physical assets, that confers the right to decide about their use (Grossman and Hart (1986)), or by institutional agreement, as in political decisions (Aghion and Tirole (1997)).
• The principal (denoted by P) chooses between centralization and partial delegation of $d_1$.
• The agent (denoted by A) observes the state of the world, $\theta$.
• Under Partial delegation,
  – A chooses $d_1$, which is observed by P.
  – P chooses $d_2$.
• Under centralization,
  – A sends a message to P.
  – P chooses $d_1$.
  – A sends a message to P.
  – P chooses $d_2$.

**Message Game** After observing the state of the world, A may reveal information by sending a message to P. This message may induce P to update her prior beliefs and, therefore, may affect her choice of the projects.\(^{10}\) We allow A to send a message before each decision taken by P. We do not allow the agent to communicate under partial delegation.

**Preferences** P and A derive private benefits from each project. The payoffs of player P and A are represented by a quasi-concave Von Neumann-Morgenstern utility function $U^P(d_1, d_2, \theta)$ and $U^A(d_1, d_2, \theta)$. We assume that utility functions satisfy:

**A1: Time-separability:** For $K = P, A$: $U^K(d_1, d_2, \theta) = U^K_1(d_1, \theta) + U^K_2(d_2, \theta)$.

**A2: Single peakedness:** For $t = 1, 2$ and $K = P, A$, there exists a unique $\hat{d}_K^t(\theta) \equiv \max_{d_t} U^K_t(d_t, \theta)$.

**A3: Divergence of interests:** For $t = 1, 2$: $\hat{d}_A^t(\theta) \neq \hat{d}_P^t(\theta)$.

**A4: Increasing difference (ID):** For $t = 1, 2$ and $K = P, A$: $\Delta U^K_t(d_t) = U^K_t(d_t, \theta_H) - U^K_t(d_t, \theta_L)$ increases in $d_t$. This assumption is a standard sorting condition in a two states of the world framework.

**Lemma 1** A2 and A4 imply that for $t = 1, 2$ and $K = A, P$, $\hat{d}_A^K(\theta_H) > \hat{d}_P^K(\theta_L)$.

**Proof.** Single peakedness implies that $U^K_t(., \theta)$ is increasing in $d_t$ for $d_t < \hat{d}_K^K(\theta)$ and decreasing for $d_t \geq \hat{d}_K^K(\theta)$. Suppose that $\hat{d}_A^K(\theta_L) > \hat{d}_P^K(\theta_H)$. Then, we have $\Delta U^K_t(d_t)$ decreasing in $d_t$ for $d_t \in (\hat{d}_A^K(\theta_H), \hat{d}_P^K(\theta_L))$ which is in contradiction with A4. \(\blacksquare\)

In our set-up, for P and A there is a unique state-contingent preferred project at each period $t$. P and A, however, disagree on which project is the best one.

**Liquidity constraints** We assume that the agent is liquidity constrained. We can interpret this assumption as follows: The agent is liquidity constrained because of imperfect capital markets and, therefore, he cannot buy the organization with a pay-cut at the beginning of the relationship. Without this hypothesis, the problem has an easy solution: P is weakly better off if both A and P can agree on an appropriate pay-cut and let A make both decisions. This\(^{10}\) makes no difference whether messages are verifiable or unverifiable, because we do not consider the case where the allocation of control rights could be contingent on messages.
assumption is quite realistic and is fairly common in the literature (see, Zabojnik (2002) and Sappington (1983)). Our Proposition 9 shows when an agent which is not liquidity constrained can improve the performance of the organization.

3 Partial delegation

This section describes the projects chosen by the agent and the principal when \( d_1 \) is delegated and \( d_2 \) is not. Under partial delegation, \( P \) observes \( A \)'s decision before choosing \( d_2 \). Given that \( A \) is better informed, his choice \( d_1 \) may have an informational content. Therefore, \( P \) revises her prior beliefs before choosing the period 2 project and \( A \) will take this into account.

We adopt the standard equilibrium concept used in signaling games, i.e., the Bayesian-Nash equilibrium (BNE).

Let \( BR^A(d_2) \) and \( BR^P(d_1) \) denote the best response correspondences of the two players.

**Definition 1** A Bayesian-Nash equilibrium (BNE) is a strategy profile for each player, \((d_1^*(.), d_2^*(.))\), such that

\[
d_1^*(.) \in BR^A(d_2^*) \equiv \arg\max_{d_1} U^A_1(d_1, \theta) + U^A_2(d_2^*, \theta), \forall \theta,
\]

\[
d_2^*(.) \in BR(d_1^*, \Theta) \equiv \arg\max_{d_2} \sum_{i=1}^{2} \mu^*(\theta_i|d_1^*)U^P_2(d_2, \theta_i),
\]

where posterior beliefs \( \mu^*(\theta_i|d_1^*) \) are consistent with Bayes’ rule.

Usually, signaling games have multiple equilibria. We use the intuitive criterion (Cho and Kreps (1987)) to restrict the equilibrium set.

**Definition 2** Let \( A \)'s equilibrium payoff in state \( \theta \) be \( U^A_*(\theta) = U^A(d_1^*, d_2^*, \theta) \). A BNE fails the intuitive criterion if, in state \( \theta_i \), \( \forall d_1 \neq d_1^* \),

\[
U^A_*(\theta_i) > \max_{d_2 \in BR(d_1, \theta)} U^A(d_1, d_2, \theta_i), \tag{1}
\]

and, in state \( \theta_j \), there is some \( d_1 \) such that

\[
U^A_*(\theta_j) < \min_{d_2 \in BR(d_1, \Theta \setminus \theta_i)} U^A(d_1, d_2, \theta_j) \tag{2}
\]

A BNE fails the intuitive criterion if \( A \)'s equilibrium payoff in one state of the world \( (\theta_i) \) is greater with the equilibrium strategy \( d_1^* \) than with any other strategy (condition (1)). Moreover, there must exist a \( d_1 \) such that \( A \)'s equilibrium payoffs in the other state of the world \( (\theta_j) \) are smaller than those with the strategy \( d_1^* \), once \( P \) is convinced that \( d_1 \) could not have been chosen by \( A \) in state \( \theta_i \) (condition (2)).

In the remaining of this section, we describe the outcome of the signaling game played by the two individuals when \( P \) delegates \( d_1 \). Results are summarized in Proposition 2.

**Proposition 2** Under partial delegation, the only equilibrium that survives the intuitive criterion is the least costly separating (LCS) equilibrium (i.e., the "Riley’s outcome"\(^{11} \)).

\(^{11}\)Riley (1979).
Bear in mind that existence of a BNE is not at issue here and, therefore, we may just focus on the informational properties of equilibria. Our strategy of proof is to consider the possible configurations of separating and pooling equilibria, and to show that only the “Riley’s outcome” survives, once we require equilibria to satisfy the intuitive criterion. First, we look at the possible separating equilibria.

**Separating equilibria** If the equilibrium is separating, by observing $d^*_i(\cdot)$, $P$ learns $\theta_i$. Hence, she selects her preferred project:

$$d^*_2(\theta) = \tilde{d}_2^P(\theta).$$  \hfill (3)

A strategy profile $(d^*_i(\cdot), d^*_2(\cdot))$ is a separating equilibrium if the following incentive compatibility constraints are satisfied:

$$U_1^A(d^*_1(\theta_L), \theta_L) + U_2^A(d^*_2(\theta_L), \theta_L) \geq U_1^A(d^*_1(\theta_H), \theta_L) + U_2^A(d^*_2(\theta_H), \theta_L)$$  \hfill (4)

$$U_1^A(d^*_1(\theta_H), \theta_H) + U_2^A(d^*_2(\theta_H), \theta_H) \geq U_1^A(d^*_1(\theta_L), \theta_H) + U_2^A(d^*_2(\theta_L), \theta_H)$$  \hfill (5)

To determine the relevant constraint, we must first identify the state where $A$ could have an incentive to misrepresent his information. This may happen in state $\theta_i$ only if the induced period 2 decision satisfies:

$$U_2^A(d^*_2(\theta_j), \theta_i) \geq U_2^A(d^*_2(\theta_i), \theta_i).$$  \hfill (6)

Indeed, $A$ has to make a non-optimal decision to misrepresent his information. This behavior can be optimal for $A$ only if $P$’s period 2 decision counterbalances his losses in the first period.

Three mutually exclusive cases must be distinguished: (S1) condition (6) is satisfied in state $\theta_L$, i.e. if $\theta_L$, $A$ has a second period benefit if he misrepresents his type, (S2) condition (6) is satisfied in state $\theta_H$, and (S3) condition (6) is neither satisfied in $\theta_L$ nor in $\theta_H$. The increasing difference assumption rules out the case where (6) would have been satisfied in both states.

Consider case S1. $A$’s benefits of misrepresenting his type in state $\theta_L$ are given by

$$U_2^A(d^*_2(\theta_H), \theta_L) - U_2^A(d^*_2(\theta_L), \theta_L) > 0.$$  \hfill (7)

By ID, if equation (6) is satisfied for $\theta_i = \theta_L$, then it is not satisfied for $\theta_i = \theta_H$. Hence, in case S1, the relevant incentive constraint is given by (4). This constraint could be equivalently written as:

$$U_1^A(d^*_1(\theta_L), \theta_L) - U_1^A(d^*_1(\theta_H), \theta_L) \geq U_2^A(d^*_2(\theta_H), \theta_L) - U_2^A(d^*_2(\theta_L), \theta_L).$$  \hfill (8)

This means that, in state $\theta_L$, $A$’s costs (in the first period) of mimicking the behavior of $A$ in state $\theta_H$ (given by the left-hand-side of equation (8)) are greater than the benefits he would have in the second period (given by equation (7)).

$P$ uses Bayes’rule to update her beliefs, hence, at a separating equilibrium, $\mu(\theta_L|d^*_1(\theta_L)) = 1$ and $\mu(\theta_L|d^*_1(\theta_H)) = 0$. In (S1), the equilibrium is supported by beliefs $\mu(\theta_L|d_1) = 1, \forall d_1 \neq d^*_1(\theta_H)$. Given $P$’s beliefs, $A$ selects his preferred project in state $\theta_L$ and the set of separating equilibria is given by:

$$d^*_i(\theta_L) = \tilde{d}_1^A(\theta_L).$$  \hfill (9)

$$d^*_i(\theta_H) \in D \equiv \{d_1(\theta_H) \mid (8) \text{ is satisfied}\}.  \hfill (10)$$

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12 In the analysis, we neglect the agent’s outside option (the individual rationality constraints) and assume that the agent’s equilibrium payoff is always higher than his outside option. Consequently, a separating equilibrium always exists. Alternatively, if the total payoff ($U^A + U^P$) is positive and if $P$ has liquidity, she can make an unconditional transfer to $A$, which is equivalent to a decrease of his outside option.
Given (7), it is easy to check that these are the only possible separating equilibria in (S1).

We now use the intuitive criterion to select a unique equilibrium in D. In state \( \theta_H \), consider a deviation by A from \( d_1^*(\theta_H) \) to \( d_1 \in D \). By definition of D, such a deviation can benefit A only in state \( \theta_H \). Therefore, the intuitive criterion imposes that the beliefs associated with \( d_1 \in D \) should be updated to \( \mu(\theta_L|d_1 \in D) = 0 \).

Hence, at \( \theta_H \), a rational agent will select his preferred decision within D. The only equilibrium surviving the intuitive criterion is the efficient separating equilibrium (the ”Riley’s outcome”). If \( \hat{d}_1^{A}(\theta_H) \in D \), the ”Riley’s outcome” is \( d_1^*(\theta_H) = \hat{d}_1^{A}(\theta_H) \). Otherwise, it is \( d_1^*(\theta_L) = \hat{d}_1^{A}(\theta_L) \) and \( d_1^*(\theta_H) = \max_{d_1 \in D} U_1^{A}(., \theta_H) \). Cases S2 and S3 are similar and therefore omitted.

**Pooling equilibria** Bear in mind that, if there is no revelation, the rationale for delegation disappears and the principal is better off making both decisions herself. We analyze pooling equilibria because it is important to stress that none of them survives the intuitive criterion. Therefore, partial delegation will always entail full revelation.

If A plays a pooling equilibrium, P does not learn any information by observing \( d_1^* \) and her period 2 choice is

\[
d_2^* = \arg\max_{d_2} v_L U_2^P (d_2, \theta_L) + v_H U_2^P (d_2, \theta_H),
\]

(11)

Given our assumptions, \( d_2^* \) is unique and \( d_2^* \in [\hat{d}_2^P (\theta_L), \hat{d}_2^P (\theta_H)] \).

As in the case of separating equilibria, we have to identify the state where A hides his information. He prefers an uninformed P in state \( \theta_i \) only if:

\[
U_2^A (d_2^*, \theta_i) \geq U_2^P (d_2^*, \theta_i).
\]

(12)

This is because, if A decides to hide his information, he has to make a sub-optimal decision in state \( \theta_i \). This behavior can be optimal only if the period 2 decision of P counterbalances A’s first period loss.

To define the out-of-equilibrium beliefs supporting the pooling equilibrium, it is important to identify the state of nature in which A has no interest in revealing his private information. Therefore, once again, we need to consider three different cases: N1 (A prefers an uninformed P in state \( \theta_L \)), N2 (A prefers an uninformed P in state \( \theta_H \)) and N3 (A always prefers an informed P).

Consider the case N1. A may prefer an uninformed P in state \( \theta_L \) only if

\[
U_2^A (d_2^*, \theta_L) \geq U_2^P (\hat{d}_2^P (\theta_L), \theta_L).
\]

(13)

Again, by ID, if this condition holds, in state \( \theta_H \), the agent prefers an informed principal, that is:

\[
U_2^A (d_2^*, \theta_H) \leq U_2^P (\hat{d}_2^P (\theta_H), \theta_H).
\]

(14)

To support the pooling equilibrium, the out-of-equilibrium beliefs must be \( \mu(\theta_L|d_1 \neq d_1^*) = 1 \).

We can now define the set of pooling equilibria as the set of \( d_1^* \) such that A’s equilibrium payoffs in both states are greater than his payoffs if he would deviate from the equilibrium. Evidently, we only have to consider deviations to A’s preferred project \( \hat{d}_1^{A}(\theta) \).

In N1, the set of pooling equilibria is the set \( d_1^* \) such that:

\[
U_1^A (d_1^*, \theta_L) + U_2^A (d_2^*, \theta_L) \geq U_1^A (\hat{d}_1^{A}(\theta_L), \theta_L) + U_2^P (\hat{d}_2^P (\theta_L), \theta_L).
\]

(15)

\[
U_1^A (d_1^*, \theta_H) + U_2^A (d_2^*, \theta_H) \geq U_1^A (\hat{d}_1^{A}(\theta_H), \theta_H) + U_2^P (\hat{d}_2^P (\theta_L), \theta_H).
\]

(16)
We now apply the intuitive criterion to get rid of all the pooling equilibria.

**Lemma 3** For each \( d_1^* \), there exists \( \tilde{d}_1 \) such that:

(i) if \( \theta_L \), \( A \) prefers the pooling equilibrium \( d_1^* \) to \( \tilde{d}_1 \), whatever the beliefs associated with \( \tilde{d}_1 \).

(ii) if \( \theta_H \), \( A \) prefers \( \tilde{d}_1 \) to the pooling equilibrium \( d_1^* \) if \( P \) believes that \( \mu(\theta_L|\tilde{d}_1) = 0 \).

The proof is in Appendix A.

Given Lemma 3, if \( \theta_L \), \( A \) will never deviate to \( \tilde{d}_1 \). Hence, according to the intuitive criterion, the beliefs associated with \( \tilde{d}_1 \) should be \( \mu(\theta_L|\tilde{d}_1) = 0 \). However, with these updated beliefs, if \( \theta_H \) realizes, \( A \) prefers to quit the pooling equilibrium (part (ii) of the Lemma). Hence, the initial equilibrium \( d_1^* \) does not survive the intuitive criterion.

Cases N2 and N3 are identical to N1 and therefore omitted.

This concludes the proof of Proposition 2, our key result: Given an appropriate equilibrium concept, partial delegation entails full revelation.

### 4 Centralization

In a centralized mechanism, the principal can still acquire information by asking the agent to communicate it. Given that the projects and the state of the world cannot be contracted for, the principal cannot reward or punish the agent if he does not transmit the true information. Hence, under centralization, communication is a cheap-talk game.

If the preferences are not time invariant, the agent could have different incentives to transmit information at different periods of time. Therefore, \( P \) could require \( A \) to communicate his private information twice, before each one of her decisions. Obviously, if \( A \) communicates the true information at \( t = 1 \), the second message game is useless.

In these message games, \( A \) sends a message to \( P \), who revises her beliefs about \( \theta \) before making her choice. Beliefs are updated according to Bayes’ rule. We represent \( P \)’s knowledge of the state of the world after the message game at period \( t \) by posterior beliefs \( M_t = (\mu_L, \mu_H) \) where \( \mu_i \), \( i = L, H \) is the belief that \( \theta = \theta_i \). We call \( M_0 = (v_L, v_H) \), the prior beliefs.

In the centralized mechanism, \( P \) maximizes her utility given her knowledge. At time \( t \), \( P \)’s utility is maximized for

\[
d_t^*(M_t) = \arg\max_{d_t} \mu_L U_t^P(d_t, \theta_L) + \mu_H U_t^P(d_t, \theta_H)
\]

(17)

Before each decision taken by \( P \) (according to (17)), \( A \) chooses his message. Centralization is, then, a four-stage game. For each period there are two stages: Communication and decision. We solve this game backward.

In a cheap talk game, there is always a babbling equilibrium, where \( P \) learns nothing. We now concentrate on the conditions that guarantee that an informative equilibrium exists. If it exists, we assume that the agent plays the strategy corresponding to this more informative equilibrium.

**Lemma 4** Given \( M_1 \), an informative equilibrium exists at \( t = 2 \) if \( \forall \theta \in \Theta \):

\[
U_2^A(d_2^*(M_1), \theta) \leq U_2^A(d_2^*(\theta), \theta).
\]

(18)
If $\exists \theta \in \Theta$ such that (18) is not satisfied, the only equilibrium is the babbling equilibrium where $M_2 = M_1$.

Proof. In state $\theta_i$, the agent has the choice between two strategies: Either he sends the same message as in state $\theta_j$ or he sends a different message. With identical messages sent by A in the two states, P does not acquire any information and $M_2 = M_1$. With different messages sent by A in the two states, P learns the true state $\theta$. If there exists a state of the world $\theta_i \in \Theta$ in which the agent prefers to keep the principal uninformed, the best strategy in that state is to send the same message as in the other state. Hence, P remains uninformed. It is only when, in both states, A prefers an informed principal - i.e., when A sends different messages in the two states- that an informative equilibrium exists. $\blacksquare$

Lemma 4 is intuitive. If, in state $\theta_i$, A prefers to keep P uninformed, he does so by mimicking his own behavior in state $\theta_j$. Hence, P cannot acquire information in the message game. Truthful communication takes place only if A wants to have an informed P in both states. In a message game, keeping P ignorant is costless for A: It just requires to send the same message in both states.

We now move to the equilibrium in the four stage game.

Proposition 5 (a) If for $M_1 = M_0$, an informative equilibrium exists at $t = 2$, an informative equilibrium exists at $t = 1$ if $\forall \theta \in \Theta$:

$$U^A_1(d^*_1(M_0), \theta) \leq U^A_1(\hat{d}^P_1(\theta), \theta).$$

If $\exists \theta \in \Theta$ such that (19) is not satisfied, the only equilibrium in the message game at $t = 1$ is the babbling equilibrium and information is delayed until $t = 2$.

(b) If for $M_1 = M_0$, there does not exist an informative equilibrium at $t = 2$, an informative equilibrium exists at $t = 1$ if $\forall \theta \in \Theta$:

$$U^A_1(d^*_1(M_0), \theta) + U^A_2(d^*_2(M_0), \theta) \leq U^A_1(\hat{d}^P_1(\theta), \theta) + U^A_2(\hat{d}^P_2(\theta), \theta).$$

If $\exists \theta \in \Theta$ such that (20) is not satisfied, the unique equilibrium in the message games is the babbling equilibrium.

Proof. As for Lemma 4. $\blacksquare$

To summarize: In the centralized mechanism, either (a) the principal acquires the true information at $t = 1$, or (b) the principal acquires the true information at $t = 2$, or (c) the principal acquires information neither at $t = 1$ nor at $t = 2$. Depending on the information she has, P implements either her preferred project $\hat{d}^P_1(\theta)$ or $\hat{d}^P_1(M_0)$. Contrarily to the case of partial delegation, under centralization P will not become informed in all circumstances. Costless communication from A to P does not guarantee that P always learns the state of the world.

5 Comparisons of organizational structures

We have just established that the information transmission process could have different results under the two organizational structures. The aim of this section is to determine if (and when) partial delegation is the optimal organizational structure for the principal.

Suppose that P can elicit A’s information in the first period with a message game. Then, centralization clearly dominates partial delegation, from her viewpoint. Hence, P keeps the control
over all the projects and implements her preferred ones. Therefore, for partial delegation to be the optimal organization, P must not be able to elicit A’s information through communication (at least in the first period).

When communication fails, P is not able to implement her preferred project neither with partial delegation nor with centralization. If she centralizes all decisions, the loss of utility comes is due to the fact that she does not have all the relevant information in her hands. With centralization, P bases her decision on her prior knowledge of the state of the world rather than on its true value. If the first decision is delegated to A, there is no problem of lack of information of the decision-maker. However, A, who decides in the first period, does not share the preferences of P. Hence, there are losses due to lack of control associated with delegation and losses due to lack of information associated with centralization. As in Dessein (2002) and Ottaviani (2000), the relative importance of these two effects will ultimately determine the optimal organizational structure.

In Proposition 2, we have shown that delegation is an instrument to induce revelation of information. We use an example to show that (1) communication may fail and (2) P may optimally transfer control over the first decision to A. This means that, when the agent does not communicate, the loss of control associated with delegation can be small enough compared to the benefits of information.

5.1 An example

Consider the following preferences for A and P: 
\[ U_A^t = \left( \alpha d_t - \frac{(\theta - d_t)^2}{2} \right) \text{ and } U_P^t = \left( \beta d_t - \frac{(\theta - d_t)^2}{2} \right) \] 
These preferences are time invariant and satisfy Assumptions A1 to A4. A’s preferred project at time \( t \) is \( \hat{d}_A^t(\theta) = (\alpha + \theta) \), while P’s preferred project is \( \hat{d}_P^t(\theta) = (\beta + \theta) \). We suppose that A has a bias toward larger projects, that is \( \alpha > \beta \).

**Partial delegation** Following Proposition 2, under partial delegation the only equilibrium surviving the intuitive criterion is the “Riley’s outcome”. To compute it in our example, we need to identify the relevant incentive constraint. In a separating equilibrium, P’s decision at \( t = 2 \) is \( d_2^t(\theta) = \hat{d}_A^t(\theta) = (\alpha + \theta) \). Since \( \alpha > \beta \), if \( \theta_H \), A prefers to signal his type. Hence, the relevant incentive constraint is then given by (4) and it defines the set of separating equilibria. Applying the intuitive criterion, the ”Riley’s outcome” is:

\[ d_1^t(\theta_L) = \hat{d}_A^t(\theta_L) = \alpha + \theta_L, \]  
\[ d_1^t(\theta_H) = \begin{cases} \hat{d}_A^t(\theta_H) = \alpha + \theta_H \\ \alpha + \theta_L + \sqrt{(2\alpha - 2\beta - \Delta\theta)\Delta\theta} > \hat{d}_A^t(\theta_H) \end{cases} \text{ if } \Delta\theta \geq \alpha - \beta, \]  
\[ d_2^t(\theta) = \hat{d}_P^t(\theta) = \beta + \theta. \]

**Centralization** In the centralized mechanism, P maximizes her utility given her knowledge. At time \( t \), given beliefs \( M_t = (\mu_L, \mu_H) \), P’s utility is maximized for \( d_1^* = (\beta_L + \mu_L \theta_L + \mu_H \theta_H) \). With time invariant preferences, either all information is transmitted at \( t = 1 \) or the messages are completely noisy. The following Lemma derives the condition for the existence of an informative equilibrium.

**Lemma 6** An informative equilibrium in the message game exists if and only if \( (\alpha - \beta) \leq \frac{\mu_L \Delta \theta}{2} \).
Proof. This condition corresponds to condition (18) of Lemma 4 for $M_1 = M_0$. ■

Comparisons If A reveals his private information in the message game, centralization domi-
ninates since it allows the principal to select her preferred project $\hat{d}_P^t(\theta)$ at both periods. If the message game is not successful, P faces the following trade-off: Either she remains non-informed and take decisions by herself or she delegates $d_1$ and both decisions are taken by an informed party (A at $t = 1$, P at $t = 2$) but, in the first period, the decision-maker is biased. Hence, the choice between centralization and partial delegation depends on the value of information vs. the losses due to lack of control.

Lemma 7 Partial delegation is preferred to uninformed centralization if

$$
(\alpha - \beta)^2 \leq 2v_L v_H \Delta \theta^2. \tag{24}
$$

Equation (24) has an easy interpretation: Its left hand side represents the loss of control, its right hand side the value of information. Loss of control can be expressed as the difference between P’s expected utility when she takes a decision knowing the true state $\theta$ and her utility when A decides: $E_{\Theta} U_P^t(d_P^t(\theta), \theta) - E_{\Theta} U_P^t(\hat{d}_P^t(\theta), \theta) = (\alpha - \beta)^2$. The value of information is the difference in P’s expected utility when her decision is based on the true state $\theta$ rather than on its prior knowledge. Without knowing $\theta$, P selects the decision $d_1^*(M_0) = (\beta + v_L \theta_L + v_H \theta_H)$. At each period $t$, information increases P’s utility by: $E_{\Theta} U_P^t(\hat{d}_P^t(\theta), \theta) - E_{\Theta} U_P^t(d_1^*(M_0), \theta) = v_L v_H \Delta \theta^2$. Hence, $(\alpha - \beta)^2$ is the lost utility due to a biased agent and $2v_L v_H \Delta \theta^2$ is the benefits of having an informed decision-maker at both periods. Note that when partial delegation is preferred to uninformed centralization, the agent selects at $t = 1$ his preferred project $\hat{d}_A^1(\theta)$.13 Summing up, we have:

Proposition 8 The principal prefers partial delegation if

$$
\frac{(\alpha - \beta)}{\sqrt{2v_L v_H}} \leq \Delta \theta \leq \frac{2(\alpha - \beta)}{v_H}. \tag{24}
$$

Partial delegation dominates when $\Delta \theta$ is not sufficiently large to have revelation of information in the message game, but sufficiently large to have the benefits of information larger than the loss of control. Figure 1 illustrates the result. In our model, the main reason for a transfer of control is not that the agent is better informed, but that information will be transmitted if he has control. Figure 1 shows how differences in the structure of A’s private information induce different organizations with different levels of success. Some could manage to acquire information at no cost, some should delegate and suffer the loss of control to acquire information and some would prefer to remain uninformed.

5.2 Complete delegation and partial delegation of $d_2$

So far, we concentrated only on two organizational structures: Partial delegation and centraliza-
tion. There are, however, two other possible allocations of the decision rights: Complete
delegation and partial delegation of $d_2$. These two structures have in common that information transmission plays no role. If we refer to the above example, it is clear that these two structures are never optimal.

Under complete delegation, the agent implements $\hat{d}_A^1(\theta)$ at the two periods. Complete
delegation then leads to a duplication of the loss of control.14 This structure is preferred to uninformed

---

13When the agent, to signal his type, does not select his preferred project, there are additional loss of control.

14The only reason why complete delegation could be optimal is that it does not entail an additional loss of control when the agent does not select his preferred project to signal his type.
centralization if \( \sum_t E\Theta U_t^P(\hat{d}_t^A(\theta), \theta) \geq \sum_t E\Theta U_t^P(d_t^P(M_0), \theta) \). In the example, this condition is equivalent to \((\alpha - \beta)^2 \leq v_Lv_H\Delta \theta^2\). Clearly, in the parameter space where complete delegation could be optimal, it is always dominated by partial delegation of \(d_1\). Similarly, partial delegation of \(d_2\) is strictly dominated by partial delegation of \(d_1\) when the message game is uninformative since the latter allows the principal to become informed. When the message game is informative, delegation of \(d_2\) is strictly dominated by centralization.

5.3 The role of limited liability constraint

Finally, suppose that the agent is not liquidity constrained and that he can buy the control over both decisions at the beginning of the relationship, that is before he learns \(\theta\). We call \(E\Theta U^K^*\) the expected payoff of player \(K = A, P\) under the preferred organizational structure. Both parties are better off if the agent buys the right to take both decisions for an amount \(\omega\) that satisfies:

\[
\sum_t E\Theta U_t^P(\hat{d}_t^A(\theta), \theta) + \omega \geq E\Theta U^{P*}
\]

(25)

\[
\sum_t E\Theta U_t^A(\hat{d}_t^A(\theta), \theta) - \omega \geq E\Theta U^{A*}
\]

(26)

We use now the functional forms of our example to derive the conditions under which the absence of liquidity constraint can make both parties better-off i.e. the conditions to have an \(\omega\) that satisfies (25) and (26). 

**Proposition 9** It is only when the preferred organizational structure is uninformed centralization that a non limited liability constrained agent can strictly increase the welfare of both parties.
Proof. (1) When the preferred organization is informed centralization, \( E U^A \ast = 2[\alpha(\beta + v_L \theta_L + v_H \theta_H) - \beta^2 / 2] \) and \( E U^P \ast = 2[\beta(v_L \theta_L + v_H \theta_H) + \beta^2 / 2] \). Replacing in (25) and (26), we have:

\[
\begin{align*}
\omega & \geq \frac{(\alpha - \beta)^2}{2} \\
\omega & \leq \frac{(\alpha - \beta)^2}{2}
\end{align*}
\]  

Clearly, with a cut-off pay of \( \omega = \frac{(\alpha - \beta)^2}{2} \), both parties are indifferent between informed centralization and an organization where the agent buys the control over the two decisions. The reason is that the total welfare does not increase when the agent buys the control.

(2) Similarly, when the preferred organization is partial delegation, if A buys the control, he cannot increase the total welfare. Hence, both parties cannot be strictly better-off.

(3) When the preferred organization is uninformed centralization, conditions (25) and (26) are equivalent to:

\[
\begin{align*}
\omega & \geq \frac{1}{2}(v_L v_H \Delta^2 \theta^2 - (\alpha - \beta)^2) \\
\omega & \leq \frac{1}{2}(v_L v_H \Delta^2 \theta^2 + (\alpha - \beta)^2)
\end{align*}
\]

These two equations define the set of \( \omega \) such that the agent can improve the utility of both parties if he buys the organization to the principal at the beginning of the relationship. ■

Notice that when the agent is not liquidity constrained the total welfare does not change with the organizational structure.

6 Conclusions

Our paper provides a new justification for delegation: In repeated interactions, the delegation of control to the agent induces full revelation of his private information. Hence, partial delegation is an instrument to elicit information within an organization. We establish this result in three steps. First, we show that information is indeed fully revealed when the agent gets control. Next, we show that message games could fail to be informative. Finally, we show that, when direct communication fails, partial delegation can be the optimal organizational structure from the point of view of the principal.

We establish this result in a model with two states of the worlds. In a more general setting, with \( N \) states of the world, the main result still holds: Only the "Riley’s outcome" survives the intuitive criterion if signals are free, i.e., if the agent’s preferred project belongs to the set of separating equilibria. When signals are costly, the intuitive criterion is not sufficient to eliminate all the pooling equilibria and the equilibrium under delegation may involve some degree of pooling. Nevertheless, with \( N \) state of the world, the argument is similar, even if there is some pooling. We show that a rationale for delegation is information revelation and, as long as there is some revelation, there are still benefits associated with delegation. In these cases, the information received by the principal is not complete, but she can still use it to make her choice in the second period. Moreover, with \( N \) states of the world, communication is noisy.

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15To eliminate the pooling equilibria, criterion such as divinity (Bank and Sobel, 1987) should be used.
16A situation that happens when not all the \( N \) types pool on the same decision.
and the principal should not expect the agent to communicate his private information in the cheap-talk game (Crawford and Sobel (1982) and Dessein (2002)). Hence, when communication fails, or is imperfect, delegation may still allow to elicit the agent’s hidden information.

Our analysis is conducted under the assumption that the principal can commit at the beginning of the game to a given organizational structure. If we relax this hypothesis, it is possible that, under partial delegation of \( d_1 \), the agent plays a pooling equilibrium in order to receive control at period 2. Hiding information at the first period could be optimal for the agent, if he can induce the principal to give him control and thereby implements his preferred project at date 2. After a pooling equilibrium at date 1, the principal leaves control over \( d_2 \) to the agent only if:

\[
E_\Theta U^2_2(d_2^1(\theta), \theta) \geq E_\Theta U^2_2(d_2^2(M_0), \theta).
\]

If this condition is met, two equilibria coexists under partial delegation of \( d_1 \): A pooling equilibrium where the agent hides his information and receives control at date 2 and the separating equilibrium described in Proposition 2. Without commitment to the organizational structure, partial delegation of \( d_2 \) can be the optimal organizational structure. In our example, it corresponds to the parameter space

\[
\sqrt{v_L v_H} \leq \Delta \theta \leq \frac{2(\alpha - \beta)}{v_H}.
\]

If the principal anticipates that the agent will play a pooling equilibrium at date 1, she will not give up control in the first period. Hence, without commitment to the organizational structure, partial delegation of \( d_2 \) can be the optimal organizational structure. In our example, it corresponds to the parameter space

\[
\frac{(\alpha - \beta)}{\sqrt{v_L v_H}} \leq \Delta \theta \leq \frac{2(\alpha - \beta)}{v_H},
\]

provided that in this region the agent plays a pooling equilibrium under partial delegation of \( d_1 \). Lack of commitment could drastically change the organizational structure for those parameters.

References


\[17\] Lack of commitment to the organizational structure makes the problem and the associated results similar to Legros (1993).

\[18\] Without commitment, a pooling equilibrium can survive the intuitive criterion because, contrary to the case with commitment, in both states, the agent has an incentive to misrepresent his type.

\[19\] When two equilibria exists, the question of which equilibrium is being played by the agent becomes crucial.
A Proof of Lemma 3

Restating Lemma 3, to each $d^*_1$, we can associate a $\tilde{d}_1$ such that:

$$U^A_1(d^*_1, \theta_L) + U^A_2(d^*_2, \theta_L) \geq U^A_1(\tilde{d}_1, \theta_L) + U^A_2(X, \theta_L), \quad (31)$$

$$U^A_1(\tilde{d}_1, \theta_H) + U^A_2(\tilde{d}_2(\theta_H), \theta_H) \geq U^A_1(d^*_1, \theta_H) + U^A_2(d^*_2, \theta_H), \quad (32)$$
where $X$ is either $d^P_2(\theta_L)$, $d^*_2$ or $\hat{d}^P_2(\theta_H)$.

Assume that there exists $\tilde{d}_1$ such that

$$U_1^A(\tilde{d}_1, \theta_H) + U_2^A(\tilde{d}^P_2(\theta_H), \theta_H) = U_1^A(d^*_1, \theta_H) + U_2^A(d^*_2, \theta_H).$$

(33)

$$\tilde{d}_1 > \hat{d}^A_1(\theta_H).$$

(34)

Given such a value of $\tilde{d}_1$, in $\theta_H$, $A$ is indifferent between the pooling equilibrium $(d^*_1, d^*_2)$ and $(\tilde{d}_1, \hat{d}^P_2(\theta_H))$. Therefore, part (ii) of the Lemma is satisfied. Given that, in $\theta_H$, $A$ prefers to signal his type, the function on the right hand side of (33) is a vertical translation of the one on the left hand side. Therefore, $\tilde{d}_1$ always exists (actually two values $\tilde{d}_1$ satisfy (33) by the single peak assumption. We select those on the right of $\hat{d}^A_1(\theta_H)$). The following figure illustrates the selection of $\tilde{d}_1$ (and shows its existence).

Figure 2: $\tilde{d}_1$ defined by equations (33) and (34).

Now, we focus on part (i). This condition is satisfied if (31) holds whatever the beliefs associated with the observation of $\tilde{d}_1$. First of all, notice that, by construction, $U_1^A(\tilde{d}_1, \theta_L) < U_1^A(d^*_1, \theta_L)$.

If the beliefs associated with $\tilde{d}_1$ are $\mu(\theta_L|\tilde{d}_1) = 1$ ($X = \hat{d}^P_2(\theta_L)$), in $\theta_L$, $A$ looses on both periods: $U_1^A(\tilde{d}_1, \theta_L) < U_1^A(d^*_1, \theta_L)$ and $U_2^A(\hat{d}^P_2(\theta_L), \theta_L) < U_2^A(d^*_2, \theta_L)$ by definition of case N1. Hence, (25) is satisfied.

If the beliefs associated with $\tilde{d}_1$ are $\mu(\theta_L|\tilde{d}_1) = v_L$ ($X = d^*_2$), (31) is also satisfied, because $U_1^A(\tilde{d}_1, \theta_L) < U_1^A(d^*_1, \theta_L)$ and $U_2^A(d^*_2, \theta_L) = U_2^A(d^*_2, \theta_L)$.

If the beliefs associated with $\tilde{d}_1$ are $\mu(\theta_L|\tilde{d}_1) = 0$ ($X = \hat{d}^P_2(\theta_H)$), (31) is satisfied for sure if $A$ looses in both periods, that is if $U_2^A(\hat{d}^P_2(\theta_H), \theta_L) < U_2^A(d^*_2, \theta_L)$.

If this last condition is not satisfied, there is as before a first period cost associated with leaving the pooling but there is also a second period benefit given by:

$$U_2^A(\hat{d}^P_2(\theta_H), \theta_L) - U_2^A(d^*_2, \theta_L) > 0.$$

(35)

To have strict preference take $\tilde{d}_1 - \epsilon$. 

19
We now use the increasing difference assumption to show that the pooling equilibrium does not survive the intuitive criterion. Since $\tilde{d}_1 > d^*_1$ and $\hat{d}_2^p(\theta_H) > d^*_2$, ID implies that:

$$U^A_1(\tilde{d}_1, \theta_H) - U^A_1(d^*_1, \theta_H) > U^A_1(\hat{d}_1, \theta_L) - U^A_1(d^*_1, \theta_L)$$

(36)

and

$$U^A_2(\hat{d}_2^p(\theta_H), \theta_H) - U^A_2(d^*_2, \theta_H) > U^A_2(\tilde{d}_2^p(\theta_H), \theta_L) - U^A_2(d^*_2, \theta_L).$$

(37)

(33) and (36) imply:

$$U^A_2(d^*_2, \theta_H) - U^A_2(\hat{d}_2^p(\theta_H), \theta_H) = U^A_1(\tilde{d}_1, \theta_H) - U^A_1(d^*_1, \theta_H) > U^A_1(\hat{d}_1, \theta_L) - U^A_1(d^*_1, \theta_L).$$

(38)

(33) and (37) imply:

$$U^A_2(d^*_2, \theta_L) - U^A_2(\hat{d}_2^p(\theta_H), \theta_L) > U^A_2(d^*_2, \theta_H) - U^A_2(\hat{d}_2^p(\theta_H), \theta_H) = U^A_1(\tilde{d}_1, \theta_H) - U^A_1(d^*_1, \theta_H).$$

(39)

(38) and (39) together imply:

$$U^A_2(d^*_2, \theta_L) - U^A_2(\hat{d}_2^p(\theta_H), \theta_L) > U^A_1(\tilde{d}_1, \theta_L) - U^A_1(d^*_1, \theta_L).$$

(40)

Rearranging the terms in this last equation, we obtain (31). Hence, our candidate $\tilde{d}_1$ satisfies both conditions of Lemma 3.