Regulation of an Open Access Essential Facility *

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Abstract

In this paper we consider the problem of regulating an open access essential facility. A vertically integrated firm owns an essential input and operates on the downstream market under the roof of a regulatory mechanism. There is a potential entrant in the downstream market. Both competitors use the same essential input to provide the final services to the consumers. The regulator designs a mechanism that guarantees financing of the essential input and adequate competition in the downstream market. We consider a regulatory mechanism that grants non-discriminatory access of the essential facility to a competitor. We show that this mechanism is welfare improving but it generates inefficient entry. That is a more efficient competitor may stay out of the market or a less efficient competitor may enter the market.

Keywords: Regulation, Railways, Network, Entry, Competition, Access charge, Asymmetric Information.

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1 Introduction

There is an increasing worldwide trend to liberalize markets and introduce competition for services that were previously operated by a monopolist. Markets like electricity, railways, telecommunication services, gas and water supply are now (or will be in the near future) open to competition. A common feature of these markets is the presence of an "essential facility" (or input). The term essential facility is used to describe a facility or infrastructure which is essential for reaching customers and/or enabling competitors to carry on their business and it is a facility that cannot be cheaply duplicated. In this scenario, all competitors, contemplating entry in this market, will have to use the same essential facility (as input) to provide these services. Hence regulating the access of essential facility is an important issue.

Allowing for competition reduces the need for regulation if, due to technical changes, the entrant or entrants can bypass the infrastructure of the monopolist. For example, it is now possible to provide telecommunication services with wireless or cable TV networks and these companies can compete with wire line networks on the telecommunication services market. However, when this sort of bypass is not possible, that is, when there is a monopoly bottleneck in the production chain due to the presence of the essential facility, competition and regulation are complements rather than substitutes.¹ Long-distance electricity transmission, high-speed railtracks, local wire line telecommunication, water supply network are still considered as natural monopolies. In these markets, allowing the access of essential facility (the bottleneck input) of the incumbent firm to competitors helps to achieve a sufficient level of competition for the final services. Moreover, financing the infrastructure is also an important concern for the regulator. Therefore, there is a two dimensional conflict between granting generous access that reduces downstream profits and the possibility of recovering the cost of the essential input. Hence, the downstream market structure is a crucial part of the regulatory environment in a market with essential facility. In this paper, we deal simultaneously with this two dimensional conflict and in particular, we consider an endogenous market structure on the downstream market.

In the theory of access pricing proposed by Laffont and Tirole (1994), the market structure is assumed to be exogenously given.² In Laffont and Tirole (1994), the access charges, designed by the regulator, aim at maximizing the total consumer surplus taking the number and the type of competitors as given. Our approach is different since in our problem the regulator specifies the access charges, to be paid by the potential entrant, without knowing the entrant’s cost

²So does the Efficient Component Pricing Rule (ECPR) approach of Baumol et al. (1982) and Armstrong (2001).
conditions. Therefore, in our problem, entry takes place after the regulator has specified the access charges. Auriol and Laffont (1993) consider the problem of firms with unknown cost competing ex-ante for a market. In their problem, the market structure, ex-post, is part of the regulatory mechanism and the regulator can allow the firms to operate as a duopoly in order to reduce their information rent. Hence, the mechanism in Auriol and Laffont (1993) completely regulates the market.\(^3\) We depart from this framework and analyze the situation in which it is not possible for the regulator to regulate the activities of the potential entrant. Consequently, there is no competition ex-ante for the market (the regulator cannot decide on the market structure ex-ante) but ex-post, there is competition on the market.

In this paper, we assume that the regulator cannot extract the entrant’s cost information. Given this lack of information, the regulator allows the entrant to use the essential facility on an open access basis provided it pays the incumbent some specified price for using the same. Open access means that any competitor that meets some pre-specified requirements (for example, technical, safety or financial fitness requirements) can get access of the essential input on a non-discriminatory basis.\(^4\) Granting a non-discriminatory access to the essential input is quite common in practice. The European Commission specifies that access fees and access conditions to the bottleneck input should be designed on a non-discriminatory basis (irrespective of the entrant’s cost). For example, in the preamble to the rail directive 2002/14/EC, it is required that: "To ensure transparency and non-discriminatory access to rail infrastructure for all railway undertakings, all the necessary information required to use the access rights are to be published in a network statement" and "The charging and capacity allocation schemes should permit equal and non-discriminatory access for all undertakings". Non-discriminatory access to the railway tracks has been advocated by a large number of competition authorities, at least for freight operators. For passenger services, access is limited to moderate competition (in the UK for example).\(^5\) Similarly, in telecommunication, non-discriminatory access of wire lines to competitors is the recommended policy. It is also quite common to have an asymmetric regulatory regime between the incumbent and the entrants. The entrants are often free to pick and choose the market and consumers while the incumbent is forced to serve all consumers. Universal service obligations often apply only to the historical operator in the market. This asymmetric treatment follows from the open-access regime: non-discrimination does not allow the regulator to impose targeted regulatory constraints on the entrant beyond some minimal

\(^3\)In Dana and Spier (1994), the government only regulates the market structure but not the firms' output.
\(^4\)In the European Union, railway undertakings need to apply for a license and a safety certificate delivered by the Member States to provide rail services.
\(^5\)Campos and Cantos (1999).
quality standard common to all entrants. While the established monopolist is under the roof of a more stringent regulatory regime, especially when the regulator finances, at least partially, the incumbent’s infrastructure.

Our paper integrates these two features: open and non-discriminatory access to the essential input and different regulatory regime for the incumbent and the competitor. We consider the problem of regulating both the service provision of an incumbent firm which is the owner of an essential input (for example, railway tracks) and the access charges to be paid by the competitors for using the incumbent’s input. Cost recovering of the bottleneck input is a major concern in this problem. This paper describes the regulatory mechanism when the essential input is operated on a non-discriminatory open access basis. It also raises the question of efficient entry. Entry is not efficient when either a more efficient competitor stays out of the market or a less efficient competitor enters the market.

The regulatory mechanism, under known marginal cost of the incumbent, prescribes (1) above marginal cost pricing if entry does not occur but a price below the incumbent’s marginal cost if entry does occur, (2) a positive lump-sum entry fee to be paid by the entrant to finance the incumbent’s network cost, (3) a per-unit subsidy to be paid by the incumbent to the entrant to reduce the post entry marginal cost of the entrant to achieve near competitive output, (4) a public subsidy to finance the infrastructure cost uncovered by access charges, (5) competition reduces the share of public transfer in infrastructure financing and (6) absence of cross subsidization of the incumbent’s network costs by the incumbent’s profit on the downstream market. Under this regulatory regime, a potential entrant, more efficient than the regulated incumbent, may stay out of the market. The regulator imposes high access charges and thereby allows less entry to reduce the financial needs of the infrastructure owner. Therefore, entry of a more efficient competitor does not necessarily occur since there is a conflict between competition and infrastructure financing. If the regulator is unaware of the incumbent’s cost of producing downstream services, she designs a mechanism which leaves an information rent to the more efficient incumbent firm. Given that these rents are socially costly, the regulator reduces these rents by allowing more entry on the downstream market. A more severe competitive pressure reduces the information rents paid to the incumbent. Hence, under asymmetric information, the regulator partially substitutes incumbent’s production by entrant’s production. However, inefficient entry occurs in both directions. An entrant less cost efficient than the incumbent can enter the downstream market or an entrant more efficient cannot operate on the market.

One can compare the results of our paper with other papers dealing with the problem of regulation with an endogenous market structure. Caillaud and Tirole (2003) consider the problem
of infrastructure financing under asymmetric information. Like in our model, an open access policy raises welfare but since competition reduces profit, the project could be non-viable if it is operated on an open access basis. However, in their model, a monopoly franchise on the downstream market is granted to the incumbent firm in situations where competition may be highly valuable, for example, when the market has a high future profitability. It is only for less favorable market conditions that the downstream market is open to competition. This pattern results from the common value environment of the model. Both social welfare and profit increase with the market profitability and it is not possible for the regulator to extract the private information of the incumbent when the market profitability is high unless it grants a monopoly franchise. The regulator and the incumbent firm are in a situation of non-responsiveness where efficiency and incentives conflict.\textsuperscript{6} Caillaud (1990) considers the problem of regulating an incumbent firm facing the threat of entry on the downstream market. Like in our setting, unregulated competitors can enter the downstream market after the regulator had specified the regulatory mechanism. But, in Caillaud (1990) competitors bypass the essential input of the incumbent and supplies the service on the downstream market using an alternate technology. The regulator then has fewer instruments to influence the entry decision and she cannot credibly deter entry by setting large access charges. Caillaud’s model applies, for example, to competition between a regulated train operator and competitive coach services. In his framework, there is a competitive fringe of entrants on the market and the regulator shuts down the incumbent if the competitors are more efficient than the incumbent. This is not possible in our framework, since shut down implies that the market would be then left to an unregulated monopolist. In a market where competitors use the incumbent’s essential facility, it is not possible to have a competitive fringe of entrants because perfect competition drives profits to zero. Hence, either the access charges should be set to zero to allow for entry or, if the regulator wants the entrants to partially finance the infrastructure, there is no entry.

2 The main problem

A regulated incumbent firm provides services to the final consumers. To provide the services, the firm uses an essential facility (network) as input. There is a potential entrant on the downstream market. To operate on the downstream market, both the incumbent and the entrant use the same essential input. The entrant can use the incumbent’s essential input provided that it

\textsuperscript{6}Guesnerie and Laffont (1984).
pays the incumbent an appropriate access charge.\textsuperscript{7} We suppose that the incumbent firm is vertically integrated and owns the network. The incumbent receives public subsidies to finance the network. Given that the public authority finances (at least partially) the essential input of the incumbent, it regulates the incumbent on the downstream market. In particular, the regulator (or public authority) specifies the amount of services that the incumbent should supply to the consumers. The regulator also specifies the amount of access charge that the incumbent should receive from the entrant if it decides to enter the downstream market. The access charge, in our problem is a two-part tariff that is, it consists of a fixed entry fee and a per-unit fee. Given the open access nature of the essential facility, an entrant who pays the access charge to the incumbent, can operate on the downstream market without being further regulated. The regulatory mechanism is observable and hence the potential entrant decides on whether or not to enter the downstream market after observing the decision imposed on the incumbent by the regulator. Figure 1 depicts the timing of the events.

Figure 1: The timing of the events

The downstream market demand function is \( P(Q) = a - bQ \) where \( a > 0 \), \( b > 0 \), \( Q \) is the quantity demanded and \( P(Q) \) is the market clearing price corresponding to \( Q \). The vertically integrated incumbent builds up the essential facility and provides services to the final consumers. The (known) fixed cost of building up the infrastructure is \( c > 0 \). There is no cost of using the infrastructure.\textsuperscript{8} The incumbent can supply services at a constant marginal cost \( \theta \in [\underline{\theta}, \overline{\theta}] \). The total cost of the integrated firm (or incumbent), when it supplies \( q_i \) units, is \( C(q_i, \theta) = \theta q_i + c \). We will consider the possibility of both known and unknown cost. When the cost is unknown, we need the following assumptions for our analysis.

1. The marginal cost has a continuous and almost everywhere differentiable density \( f(\cdot) \) and \( f(\theta) > 0 \) for all \( \theta \in [\underline{\theta}, \overline{\theta}] \).

\textsuperscript{7}The European Union adopted this type of regime for gas, electricity and rail markets: the network owner should provide access to competitors on a non-discriminatory basis.

\textsuperscript{8}This is for simplicity only. If there is a marginal cost \( \psi \) of operating on the network, the per-unit access charge paid by the entrant and the incumbent’s marginal cost should both be increased by \( \psi \) and the analysis is identical.
2. The distribution satisfies the hazard rate condition: \( \frac{F'(\theta)}{f(\theta)} \) is increasing in \( \theta \).

The potential entrant can produce an amount \( q_e \) at a marginal cost of \( \phi \in [\theta, \bar{\theta}] \). Throughout
the paper we assume that the marginal cost of the entrant is unknown and that it follows a
uniform distribution, that is \( g(\phi) = \frac{1}{\Delta} \) for all \( \phi \in [\theta, \bar{\theta}] \) where \( \Delta \equiv \bar{\theta} - \theta \). We also assume that
the marginal cost of the entrant is independent of the marginal cost of the incumbent. When
correlation between the incumbent’s and the entrant’s cost is positive, the regulatory problem
is in general non concave, hence untractable (see Caillaud (1990)). With independence, the
problem may be non concave too but a solution is possible. The entrant observes the incumbent’s
regulatory regime before his entry decision. If the potential entrant decides to enter the market,
it has to compensate the incumbent adequately. We consider a two part access charge—a fixed
fee \( A \) and a per-unit fee \( \alpha \). In our problem, an entrant, that supplies \( q_e > 0 \) units of the facility,
pays \( A + \alpha q_e \) to the incumbent. The amounts of \( A \) and \( \alpha \) are specified by the regulator.

The regulator maximizes the consumer surplus net of transfer to the incumbent, that is,
\( W = S(Q) - P(Q)Q - t \) where \( S(Q) = \int_0^Q P(x)dx \) and \( t \) is the amount of transfer paid by the
regulator to the incumbent. In this context, the transfer \( t \) is meant to finance the infrastructure.
We assume that there exists a non-distortionary tax system and hence the shadow cost of public
fund is equal to zero. Hence, the regulator has four instruments at her disposal—the quantity of
the incumbent \( q_i \), the transfer \( t \), the fixed fee \( A \) and the per-unit fee \( \alpha \).

There are two stages to our problem. In the first stage the regulator offers a regulatory
mechanism to the incumbent. The regulatory mechanism \( M = \langle q_i(\cdot), A(\cdot), \alpha(\cdot), t(\cdot) \rangle \) specifies a
quantity-access charge-transfer quadruple that depends on the marginal cost of the incumbent.
Here the quantity of the incumbent, fixed fee and per unit fee of the entrant and the transfer are
mappings from the interval of types to a subset of the real line. In particular, \( q_i : [\theta, \bar{\theta}] \rightarrow \mathbb{R}_+ \),
\( A : [\theta, \bar{\theta}] \rightarrow \mathbb{R} \), \( \alpha : [\theta, \bar{\theta}] \rightarrow \mathbb{R} \) and \( t : [\theta, \bar{\theta}] \rightarrow \mathbb{R} \). We restrict attention to continuous and
differentiable mechanisms. Given the regulatory mechanism, in stage 2, the potential entrant
decides whether or not to enter the market. If it decides to enter, it pays the access charge to
the incumbent and decides what amount of service it will supply.

In our framework, the entrant observes the regulatory mechanism before it makes its entry
and quantity decisions. The entrant is a Stackelberg follower in the quantity game. The price is
then set to equate demand and supply. The Stackelberg structure of the quantity game reflects
the dominant position of the incumbent in the downstream market. Competitive entry is not
possible in this framework since under perfect competition, the price is driven to marginal cost
and firms make zero profit. Hence the entrant cannot pay the access charges. Before analyzing
our main problem, we first study the case of no entry under both complete and incomplete information.

3 Downstream Monopoly

In this section we assume that the incumbent firm does not face the threat of entry, that is the incumbent is a monopolist. The regulatory mechanism \( \tilde{M}(\theta) = \langle q_i(\theta), t(\theta) \rangle \) then specifies a quantity transfer pair only. If the regulator knows the true cost of the incumbent, the optimal regulatory mechanism is obtained by maximizing \( W(\theta) = S(q_i(\theta)) - P(q_i(\theta))q_i(\theta) - t(\theta) \) subject to the participation constraint of the incumbent: \( \Pi_i(\theta) = (P(q_i(\theta)) - \theta)q_i(\theta) - c + t(\theta) \geq 0 \).

**PROPOSITION 3.1** Without the threat of entry, the optimal regulatory mechanism under complete information is \( \tilde{M}(\theta) = \langle q_i^f(\theta), t^f(\theta) \rangle \) for any given \( \theta \in [\underline{\theta}, \overline{\theta}] \) where

1. \( q_i^f(\theta) = \frac{a-\theta}{b} \) and
2. \( t^f(\theta) = c \)

Observe that given the objective function of the regulator and the participation constraint of the monopolist, it is optimal for the regulator to set the profit of the monopolist to zero. Incorporating profit equals to zero in the objective function we get the optimal regulatory quantity and hence the transfer. Under the optimal mechanism, the regulated downstream monopolist is required to produce the quantities \( q_i^f(\theta) \) such that the market clearing price equals the firm’s marginal cost: \( P(q_i^f(\theta)) = \theta \). Consumer surplus is then maximized but the firm cannot cover its infrastructure cost with its receipts. Hence, the regulator fully finances the infrastructure with transfer: \( t^f(\theta) = c \).

If the regulator is unaware of the true marginal cost of the monopolist, the regulator’s objective now is to design a direct mechanism that maximizes the expected welfare \( \int_{\underline{\theta}}^{\overline{\theta}} W(\theta)f(\theta)d\theta \) subject to the participation constraint and incentive compatibility constraint of the monopolist.\(^9\) Using the Revelation Principle we restrict our attention to direct revelation mechanism where the firm announces a type (or marginal cost) and based on this announcement the mechanism \( \tilde{M}(\theta) = \langle q_i(\theta), t(\theta) \rangle \) specifies a type contingent quantity-transfer pair. Let \( \Pi_i(\theta'; \theta) = \Pi_i(\theta') - (\theta - \theta')q_i(\theta') \). In other words, \( \Pi_i(\theta'; \theta) \) is the profit of the incumbent firm, under the mechanism \( \tilde{M}(\theta) \), if the true marginal cost is \( \theta' \in [\underline{\theta}, \overline{\theta}] \) and if the announcement is \( \theta' \in [\underline{\theta}, \overline{\theta}] \). Clearly, \( \Pi_i(\theta; \theta) = \Pi_i(\theta), \forall \theta \in [\underline{\theta}, \overline{\theta}] \). Given these definitions, incentive

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\(^9\) The problem of regulating a monopolist with unknown cost was first considered by Baron and Myerson (1982).
compatibility constraint states that \( \Pi_i(\theta) \geq \Pi_i(\theta'; \theta''), \forall (\theta, \theta') \in [\theta, \theta']^2 \) and, as stated earlier, \( \Pi_i(\theta) \geq 0, \forall \theta \in [\theta, \theta'] \) is the participation constraint. It is well known in the literature that the optimal mechanism \( \tilde{M}(\cdot) = \langle q_i^\ast(\cdot), t(\cdot) \rangle \) satisfies the incentive compatibility and the participation constraint if and only if \( \forall \theta \in [\theta, \theta'] \), the optimal quantity \( q_i^\ast(\theta) \) is non-increasing in \( \theta \) and \( \Pi_i(\theta) = \int_0^\theta q_i^\ast(\tau)d\tau \). The objective of the regulator is to select that \( \tilde{M}(\cdot) = \langle q_i(\cdot), t(\cdot) \rangle \) (from the class of direct revelation mechanisms) which maximizes \( \tilde{W} \equiv \int_\theta^{\theta=\theta} [S(q_i(\theta)) - P(q_i(\theta))q_i(\theta) - t(\theta)] f(\theta)d\theta \) subject to (i) \( \Pi_i(\theta) \geq \Pi_i(\theta'; \theta''), \forall (\theta, \theta') \in [\theta, \theta']^2 \) and (ii) \( \Pi_i(\theta) \geq 0, \forall \theta \in [\theta, \theta'] \).

Before providing the solution we define \( L(\theta) = \frac{F(\theta)}{f(\theta)} \) as the hazard function and \( z(\theta) = \theta + L(\theta) \) as the virtual type function. \( z(\theta) \) is increasing in \( \theta \) by assumption 2.

**PROPOSITION 3.2** Without the threat of entry, the optimal regulatory mechanism under incomplete information is \( \tilde{M}(\cdot) = \langle q_i^\ast(\cdot), t(\cdot) \rangle \) where for any announced \( \theta \in [\theta, \theta'] \),

1. \( q_i^\ast(\theta) = q_i(\theta) = \frac{a - z(\theta)}{b} \) and
2. \( t(\cdot) = c - L(\theta)q_i^\ast(\theta) + \int_0^{\theta=\theta} q_i^\ast(\tau)d\tau \)

The proof is fairly standard in the mechanism design literature and is hence eliminated. Here the market clearing price equals the virtual marginal cost \( z(\theta) \) instead of the true marginal cost \( \theta \). Observe that the relevant constraint is not the participation constraint but the incentive constraint (for all but the highest type). The firm truthfully reveals its marginal cost only if it gets adequate information rent. To reduce this cost of truth-telling, the regulator optimally distorts the quantity supplied by the firm and hence the price is above the marginal cost for all but the lowest cost firm. Once the optimal amount of service is specified, the transfer of the monopolist with type \( \theta \) follows from \( \Pi_i(\theta) = \int_0^{\theta=\theta} q_i^\ast(\tau)d\tau \). Given the form of the optimal quantity, \( z(\theta) \) is increasing in \( \theta \) is necessary to guarantee that the optimal quantity is decreasing in \( \theta \). This in turn is a sufficient condition to guarantee that incentive compatibility and individual rationality conditions are satisfied. Thus, the assumption on \( L(\theta) \) guarantees a separating optimal mechanism.

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\(^{10}\)The reason why optimal quantity is non-increasing follows directly by solving the inequality in the definition of incentive compatibility for any pair of types. The reason for the necessity of \( \Pi_i(\theta) = \int_0^{\theta=\theta} q_i^\ast(\tau)d\tau \) is the following. For incentive constraint to hold, it is necessary that \( \frac{\partial \Pi_i(\theta)}{\partial \theta} = -q_i^\ast(\theta) \) for almost all \( \theta \in [\theta, \theta'] \). Moreover, under the optimal mechanism participation constraint implies that \( \Pi_i(\theta) = 0 \). These two conditions together imply that \( \Pi_i(\theta) = \int_0^{\theta=\theta} q_i^\ast(\tau)d\tau \) for all \( \theta \in [\theta, \theta'] \).
4 Downstream Competition

In this section, we describe the regulatory mechanism when there is the possibility of entry in the downstream market. We consider two sub-cases, one where the cost of the incumbent in known and one where it is unknown.

4.1 Known cost of the incumbent

In the presence of entry possibility, the regulatory mechanism $M(\theta)$ specifies a quadruple: $M(\theta) = \langle q_i(\theta), A(\theta), \alpha(\theta), t(\theta) \rangle$ for any given and commonly known marginal cost $\theta \in [\underline{\theta}, \bar{\theta}]$ of the incumbent. The entry decision is taken after the regulator has designed the regulatory mechanism. Therefore, even if the entrant is not regulated, the regulatory mechanism (except the transfer received by the incumbent) affects the entry decision and the quantity supplied by the entrant. For brevity of notation let us denote $\langle q_i(\theta), A(\theta), \alpha(\theta) \rangle$ by $V(\theta)$. The entrant’s maximization problem is to select $q_e(\theta, V(\theta)) = \max_{q_e} \Pi_e(q_e, V(\theta); \phi)$ where $\Pi_e(q_e, V(\theta); \phi) \equiv (P(q_e + q_i(\theta)) - \alpha(\theta) - \phi)q_e - A(\theta)$. Assuming entry takes place, the solution to this problem is

$$q_e(\theta, V(\theta)) = \frac{a - \alpha(\theta) - \phi}{2b} - \frac{q_i(\theta)}{2}$$

The profit of an entrant, active in the downstream market, is:

$$\Pi_e(q_e(V(\theta)), V(\theta); \phi) = \frac{(a - \alpha(\theta) - \phi - bq_i(\theta))^2}{4b} - A(\theta)$$

Clearly, the potential entrant will enter the downstream market if its profit is strictly positive. Given the regulatory mechanism $M(\theta) = \langle q_i(\theta), A(\theta), \alpha(\theta), t(\theta) \rangle$, it is obvious from (4.2) that the potential entrant enters the downstream market if $\phi \in [\underline{\theta}, K(\theta))$ where $K(\theta) = P(q_i(\theta)) - \alpha(\theta) - 2\sqrt{bA(\theta)}$. It is also clear from (4.2) that the larger the gap between the no-entry price $P(q_i(\theta)) = a - bq_i(\theta)$ and the entrant’s marginal cost plus the per-unit fee $\phi + \alpha(\theta)$, the larger the entrant’s profit. Hence, there is entry if the price marginal cost gap (including in the marginal cost the entry fee $\alpha(\theta)$) is sufficiently large to cover the fixed entry cost $A(\theta)$. Therefore, the profit of the entrant, with per unit cost $\phi \in [\underline{\theta}, \bar{\theta}]$, is

$$\Pi_e(q_e(V(\theta)), V(\theta); \phi) = \begin{cases} 0 & \text{if } \phi \geq K(\theta) \\ (P(Q(\theta)) - \alpha(\theta) - \phi)q_e(V(\theta)) - A(\theta) & \text{if } \phi < K(\theta) \end{cases}$$

where $Q(\theta) = q_i(\theta) + q_e(V(\theta))$. The ex-post profit of the incumbent under the mechanism $M(.)$ with known marginal cost $\theta \in [\underline{\theta}, \bar{\theta}]$, is

$$\Pi_i(M(\theta); \theta) = \begin{cases} (P(q_i(\theta)) - \theta)q_i(\theta) - c + t(\theta) & \text{if } \phi \geq K(\theta) \\ (P(Q(\theta)) - \theta)q_i(\theta) - c + t(\theta) + \alpha(\theta)q_e(V(\theta)) + A(\theta) & \text{if } \phi < K(\theta) \end{cases}$$
We now consider the mechanism design problem of the regulator. Consider any mechanism $M(\cdot)$. Given that the marginal cost of the potential entrant is unknown and follows a uniform distribution over the relevant marginal cost interval $[\theta, \overline{\theta}]$, the expected profit of the incumbent in stage 1 is $\Pi_i(\theta) = \frac{1}{\Delta} \int_{\phi=\theta}^{\phi=\overline{\theta}} \Pi_i(M(\theta); \theta) d\phi$. Simplifying the expected profit of the incumbent we get

$$\bar{\Pi}_i(\theta) = \frac{1}{\Delta} \int_{\phi=\theta}^{\phi=\overline{\theta}} \{P(Q(\theta))q_i(\theta) + \alpha(\theta)q_e(V(\theta)) + A(\theta)\} d\phi + \int_{\phi=\theta}^{\phi=\overline{\theta}} P(q_i(\theta))q_i(\theta) d\phi - c - \theta q_i(\theta) + t(\theta) \quad (4.3)$$

The regulator’s objective is (I) to select a quadruple $M(\theta) = \langle q_i(\theta), A(\theta), \alpha(\theta), t(\theta) \rangle$, for any commonly known $\theta \in [\theta, \overline{\theta}]$, that maximizes $W(\theta) = \frac{1}{\Delta} \int_{\phi=\theta}^{\phi=\overline{\theta}} \{S(Q(\theta)) - P(Q(\theta))Q(\theta) - t(\theta)\} d\phi$ subject to the participation constraint of the incumbent (that is, subject to $\hat{\Pi}_i(\theta) \geq 0$).

**PROPOSITION 4.3** The solution to the regulator’s optimization problem (I) is the following: for any known $\theta \in [\theta, \overline{\theta}]$, the optimal mechanism is $M^f(\theta) = \langle q_i^f(\theta), A^f(\theta), \alpha^f(\theta), t^f(\theta) \rangle$ where

1. $q_i^f(\theta) = \frac{a - H(\theta)}{b}$
2. $\alpha^f(\theta) = -\frac{5}{11}(H(\theta) - \theta) < 0$
3. $A^f(\theta) = \frac{36(H(\theta) - \theta)^2}{121b}$ and
4. $t^f(\theta) = c - \frac{1}{\Delta} \int_{\phi=\theta}^{\phi=\overline{\theta}} \{A^f(\theta) + \alpha^f(\theta)q_e(V^f(\theta))\} d\phi = c - \frac{4(H(\theta) - \theta)^3}{11\Delta b^2} \leq c$

$H(\theta)$ is a strictly increasing and strictly convex function in $\theta$ with $H(\theta) = \theta$ and $H(\theta) > \theta$ for all $\theta \in (\theta, \overline{\theta}]$.

**PROOF:** See Appendix. $\blacksquare$

Compared to downstream monopoly where, under symmetric information, the market clears at marginal cost, here, the market price is above the incumbent’s marginal cost if entry does not occur that is $P(q_i^F(\theta)) = H(\theta) > \theta$ for all $\theta \in (\theta, \overline{\theta}]$. This means that the incumbent produces less compared to the regulated monopoly regime. However, if entry takes place, the market price falls below the incumbent’s marginal cost. Interestingly, the expected price is such that the incumbent realizes a zero profit from its downstream operations, that is $\hat{P}^f(\theta) = \frac{1}{\Delta} \left[ \int_{\phi=\theta}^{\phi=K^f(\theta)} P(q_e(\overline{V}(\theta)) + q_i^f(\theta)) d\phi + \int_{\phi=K^f(\theta)}^{\phi=\overline{\theta}} P(q_i^f(\theta)) d\phi \right] = \theta$ for all $\theta \in [\theta, \overline{\theta}]$.

Even though the expected price (the expected consumer surplus) is identical to the price (the consumer surplus) under the regulated monopoly regime, competition increases welfare because the financial contribution of the regulator is lower $(t^f(\theta) < c = \bar{t}^f(\theta)$ for all $\theta \in (\theta, \overline{\theta})$).

The infrastructure is financed by (i) the net payment of the entrant, (ii) the public transfer of the regulator. If the potential entrant decides to enter, it has to pay a lump-sum access charge
to the incumbent and it receives a per-unit subsidy from the incumbent. The reason for per-unit subsidy is to reduce the market power of the entrant. In the absence of per-unit subsidy, the entrant, being unregulated, enjoys market power and supplies less than the competitive output. In other words, in the absence of per-unit subsidy, the entrant will supply an output less than that under perfect competition (since Stackelberg follower output is lower than the competitive output). By setting \( \alpha(\theta) < 0 \) for all \( \theta \in (\underline{\theta}, \overline{\theta}) \), the regulator artificially reduces the entrant’s marginal cost and thereby partially offsets the negative effect of the entrant’s market power. It is worth noting that, if the downstream market was more competitive, that is, if there were a larger number of potential entrants, then the entrants would have had less market power and hence the regulator will subsidize them less. Being distortionary, the per-unit access charge is used only to cancel partially the entrant’s market power. Conversely, the non-distortionary fixed fee is used by the regulator to transfer profit from the unregulated competitor to the incumbent since the incumbent owns the essential facility that needs financing.

In our problem, even if the subsidy reduces the market power of the entrant, in the optimal mechanism, there is a positive net transfer from the entrant to the incumbent. This is captured by

\[
\frac{1}{\omega} \int_{\phi=\underline{\theta}}^{\phi=\overline{\theta}} \{A^f(\theta) + \alpha^f(\theta)q_e(V^f(\theta))\} d\phi = \frac{4(H(\theta) - \theta)^3}{\Pi + \Delta^2} > 0 \quad \text{for all } \theta \in (\underline{\theta}, \overline{\theta}).
\]

The fixed part of the access charge aims to partially finance the infrastructure and to deter entry of high marginal cost potential entrants. Entry needs to be restricted optimally because the price is above marginal cost under the optimal mechanism if the incumbent stays as a monopolist. This price mark up attracts inefficient entrants in the downstream market. To exclude these potential inefficient entrants, the regulator fixes a high access charge \( A^f(\theta) \). Finally, when will entry take place? The next proposition answers this question.

**Proposition 4.4** Under \( M^f(\theta) = \langle q^f_1(\theta), A^f(\theta), \alpha^f(\theta), t^f(\theta) \rangle \), entry is inefficient. Entry takes place for all \( \phi \in [\underline{\theta}, K^f(\theta)] \) where \( \underline{\theta} < K^f(\theta) < \theta, \forall \theta \in (\underline{\theta}, \overline{\theta}) \) and \( K^f(\theta) = \overline{\theta} \).

**Proof:** Under the optimal mechanism \( M^f(\theta) = \langle q^f_1(\theta), A^f(\theta), \alpha^f(\theta), t^f(\theta) \rangle \), the entry limit is \( K^f(\theta) = \left\lfloor \frac{4}{\Pi} H(\theta) + \frac{1}{\Pi} \right\rfloor \). Using \( H(\theta) \), we get \( K^f(\theta) = \theta + \frac{11\Delta}{14} - \frac{1}{11} \sqrt{121\Delta^2 - 11\Delta(\theta - \underline{\theta})} \). From \( K^f(\theta) \), it follows that \( K^f(\underline{\theta}) = \theta \) and \( K^f(\overline{\theta}) = \frac{4}{\Pi} \overline{\theta} + \frac{5}{\Pi} \theta < \overline{\theta} \). To show that \( K^f(\theta) < \theta \) for all \( \theta \in (\underline{\theta}, \overline{\theta}) \), we consider \( \theta - K^f(\theta) = (\theta - \theta) - \frac{11\Delta}{14} + \frac{1}{11} \sqrt{121\Delta^2 - 11\Delta(\theta - \theta)} \). We then apply proof by contradiction to show that \( \theta - K^f(\theta) > 0 \). Finally, \( \theta < K^f(\theta) \) for all \( \theta \in (\underline{\theta}, \overline{\theta}) \) follows from the fact that \( K^f(\theta) = \theta \) and from the fact that \( \frac{\partial K^f(\theta)}{\partial \theta} > 0 \) over the relevant range.

\(^{11}\)This result is standard in the access pricing literature. See Laflont and Tirole (2000), chapter 2. Note that with a positive marginal cost \( \psi \) of operating on the network, the optimal per-unit access charge is \( \psi + \alpha(\theta) < \psi \).

In this case, the access charge is not necessarily negative but entry is subsidized as the entrant pays less than the marginal cost of operating the service.
To preserve financing of the essential facility, the regulator bans those entrants that are more efficient than the operating incumbent but for which the cost advantage is not that high (that is, entrants whose marginal cost $\phi \in (K f(\theta), \theta]$). By limiting entry, the regulator raises the incumbent’s revenue. However, all type of incumbent firms, but the most efficient one, face a positive probability of entry since $\theta < K f(\theta)$, $\forall \theta \in (\theta, \theta]$. This means that, except for the most efficient incumbent, the regulator never grants a monopoly franchise to the incumbent firm.

When the infrastructure cost is partially financed by public transfers, it is always efficient to allow for the entry of a more efficient competitor, provided that the efficiency advantage of the competitor is sufficiently large. This part of the result is in sharp contrast to the result in Caillaud and Tirole (2003). In Caillaud and Tirole (2003), the regulator bans entry of competitors on market with high future profitability.

4.2 Unknown cost of the incumbent

In this sub-section we assume that marginal cost of the incumbent is private information. Hence, the regulator is unaware of the marginal cost of both the incumbent and the entrant. Therefore, we have a mechanism design problem of the regulator under incomplete information.

The objective of the regulator now is \( (II) \) to select \( M = (q_i(\theta), A(\theta), \alpha(\theta), t(\theta)) \), that maximizes \( \hat{W} = \int_{\theta=\theta}^{\theta=\theta} W(\theta) d\theta = \int_{\theta=\theta}^{\theta=\theta} \left[ \frac{1}{\phi} \int_{\phi=\theta}^{\phi=\theta} \{ S(Q(\theta)) - P(Q(\theta))Q(\theta) - t(\theta) \} d\phi \right] d\theta \) subject to (i) the participation constraint \( \hat{\Pi}_i(\theta) \geq 0, \forall \theta \in [\theta, \theta] \) and (ii) the incentive compatible constraint: \( \hat{\Pi}_i(\theta) \geq \hat{\Pi}_i(\theta; \theta'), \forall (\theta, \theta') \in [\theta, \theta]^2 \) where \( \hat{\Pi}_i(\theta; \theta') = \hat{\Pi}_i(\theta') - (\theta - \theta')q_i(\theta') \). Applying arguments similar to the no-entry case under asymmetric information we get that the optimal mechanism \( M \) satisfies the incentive compatibility and the participation constraint if and only if \( \forall \theta \in [\theta, \theta] \), the optimal quantity \( q_i(\theta) \) is non-increasing in \( \theta \) and \( \hat{\Pi}_i(\theta) = \int_{\theta}^{\theta} q_i(\tau) d\tau \).

The optimization problem (II) may turn out to be non concave for certain density function \( f(\cdot) \) of marginal cost of the incumbent. For example, if \( f(\cdot) \) is uniform, the optimization problem is not concave. When (II) is not concave, a characterization of the solution is possible by bunching types appropriately. The problem may be non concave because the market structure is endogenous. Though the welfare function is a regular quadratic function but since \( q_i(\theta) \) determines the entry condition, the expected surplus is a cubical equation.\(^{12}\)

\(^{12}\)Caillaud (1990) demonstrates that when a competitive fringe can compete with a regulated monopolist, in his case without using the monopolist’s essential facility, the problem may not satisfied concavity, even if the virtual type function \( z(\theta) \equiv \theta + L(\theta) \) is increasing in \( \theta \).
\((q^*_s(\theta), A^*_s(\theta), \alpha^*_s(\theta), t^*_s(\theta))\), where

1. the optimal type contingent quantity of the incumbent is

\[
q^*_s(\theta) = \begin{cases} 
q^f(z(\theta)) & \forall \theta \in [\theta, \tilde{\theta}] \\
q^f(z(\tilde{\theta})) & \forall \theta \in (\tilde{\theta}, \bar{\theta}] 
\end{cases}
\]

2. the optimal incumbent type contingent fixed access charge for the potential entrant is

\[
A^*_s(\theta) = \begin{cases} 
A^f(z(\theta)) & \forall \theta \in [\theta, \tilde{\theta}] \\
A^f(z(\tilde{\theta})) & \forall \theta \in (\tilde{\theta}, \bar{\theta}] 
\end{cases}
\]

3. the optimal incumbent type contingent per unit access charge for the potential entrant is

\[
\alpha^*_s(\theta) = \begin{cases} 
\alpha^f(z(\theta)) & \forall \theta \in [\theta, \tilde{\theta}] \\
\alpha^f(z(\tilde{\theta})) & \forall \theta \in (\tilde{\theta}, \bar{\theta}] 
\end{cases}
\]

4. the optimal type contingent transfer to the incumbent is

\[
t^*_s(\theta) = \begin{cases} 
t^f(z(\theta)) - L(\theta)q^*_s(\theta) + \int_{\theta}^{\tilde{\theta}} q^*_s(\tau)d\tau & \forall \theta \in [\theta, \tilde{\theta}] \\
t^*(\tilde{\theta}) & \forall \theta \in (\tilde{\theta}, \bar{\theta}] 
\end{cases}
\]

and

5. the full separability (that is \(\tilde{\theta} = \bar{\theta}\)) or partial separability of optimal output of the incumbent is determined by

\[
\tilde{\theta} = \begin{cases} 
\bar{\theta} & \text{if } z(\bar{\theta}) \leq \bar{\theta} + \frac{9\Delta}{112} \\
z^{-1}\left(\bar{\theta} + \frac{9\Delta}{112}\right) & \text{otherwise}
\end{cases}
\]

**PROOF:** See Appendix.

The mechanism under asymmetric information is similar to the complete information framework, modulo the facts that the regulator uses the virtual marginal cost \(z(\theta)\) for the incumbent rather than its true cost \(\theta\) and that there could be bunching of the less efficient types of incumbent.

Like in Caillaud (1990), the regulator substitutes the production of the incumbent by the production of the entrant, hence, the rents paid to the incumbent are lower while the expected price remains identical \((\hat{P}^s(\theta) = z(\theta) \text{ for all } \theta \leq \tilde{\theta})\). Thus, competition is welfare enhancing. However, the incumbent always produces while in Caillaud (1990), the incumbent is shut down when its virtual marginal cost is larger than the expected cost of the entrant. The reason is that
incumbent’s presence enhances competition on the downstream market which is not necessary in Caillaud’s framework due to the presence of a competitive fringe of entrants.

When concavity is not satisfied, that is for some \( f(\theta) \) and \( \theta > \tilde{\theta} \), the mechanism is a third best one where there is bunching. Bunching appears when there is a lot of entry i.e. for the highest values of \( \theta \). To overcome the existence problem and to preserve competition, the regulator sets a minimal quantity for the higher types.

Without bunching, the expected price \( \hat{P}^s(\theta) \) is equal to the virtual marginal cost \( z(\theta) \). Hence, for each \( \theta \), the consumer surplus is in average identical to the regulated monopoly regime under asymmetric information. Competition is welfare improving because the contribution of the regulator to the infrastructure cost is lower: \( t^*(\theta) < t^s(\theta) \). Like in the known cost case, the positive contribution of the entrant to the infrastructure cost implies a reduction of the transfer. In addition, given that the incumbent produces less when it faces the threat of entry, the information rent is also lower, hence another reason to reduce the transfer. In the bunching region, \( \hat{P}^s(\theta) = z(\tilde{\theta}) < z(\theta) \) for all \( \theta \in (\tilde{\theta}, \bar{\theta}] \). Consumer surplus in average increase compared to the monopoly regime, and competition is also welfare improving, even if transfer may increase.

In this mechanism, entry occurs if \( \phi \leq K^s(\theta) = K^f(z(\theta)) \). Again, we raise the question of efficient entry and we compare the entry levels under complete and incomplete information.

**PROPOSITION 4.6** Under \( M^s(\theta) = (q^s(\theta), A^s(\theta), \alpha^s(\theta), t^s(\theta)) \), the following can be said about the entry limit \( K^s(\theta) \).

1. If \( \tilde{\theta} = \bar{\theta} \), then \( K^s(\theta) = \theta \), \( K^s(\theta) < \bar{\theta} \), \( K^s(\theta) \) is strictly increasing in \( \theta \in (\tilde{\theta}, \bar{\theta}) \) and \( K^f(\theta) < K^s(\theta) \) for all \( \theta \in (\tilde{\theta}, \bar{\theta}] \).

2. If \( \tilde{\theta} < \bar{\theta} \), then \( K^s(\theta) = \theta \), \( K^s(\theta) < \bar{\theta} \), \( K^s(\theta) \) is strictly increasing in \( \theta \in (\tilde{\theta}, \bar{\theta}) \), \( K^s(\theta) = K^s(\bar{\theta}) \) for all \( \theta \in (\tilde{\theta}, \bar{\theta}] \) and \( K^f(\theta) < K^s(\theta) \) for all \( \theta \in (\tilde{\theta}, \bar{\theta}] \).

**PROOF:** If \( \tilde{\theta} = \bar{\theta} \), then \( K^s(\theta) = K^f(z(\theta)) = \theta \). Observe that \( K^s(\theta) = \frac{1}{11}\bar{\theta} + \frac{3}{11}\theta - \frac{1}{11}\sqrt{9\Delta^2 - 112L(\theta)} < \bar{\theta} \) since \( \sqrt{9\Delta^2 - 112L(\theta)} \) is a positive real number. Moreover, since \( K^s(\theta) = K^f(z(\theta)) \) and \( K^f(z(\theta)) \) and \( z(\theta) \) are increasing in \( z(\theta) \) and \( \theta \) respectively, it follows that \( K^s(\theta) \) is increasing in \( \theta \in (\tilde{\theta}, \bar{\theta}) \). Finally, \( K^s(\theta) - K^f(\theta) = \frac{4}{11}(H(z(\theta)) - H(\theta)) > 0 \) for all \( \theta \in (\tilde{\theta}, \bar{\theta}] \).

If \( \tilde{\theta} < \bar{\theta} \), then \( K^s(\theta) = \theta \). Given \( H(z(\tilde{\theta})) = \frac{121\Delta}{200} \), we get \( K^s(\tilde{\theta}) = \frac{11}{11}\tilde{\theta} + \frac{3}{11}\theta \). Hence, \( K^s(\tilde{\theta}) < \bar{\theta} \). Moreover, for all \( \theta \in (\tilde{\theta}, \bar{\theta}) \), \( K^s(\theta) = K^f(z(\theta)) \) and \( K^f(z(\theta)) \) and \( z(\theta) \) are increasing in \( z(\theta) \).

\(^{13}\)Note that it is not obvious whether \( K^s(\theta) \) is increasing at a decreasing or an increasing rate.
and \( \theta \) respectively. Therefore, \( K^s(\theta) \) is increasing in \( \theta \in (\bar{\theta}, \bar{\theta}) \). With bunching, \( q_i^s(\theta) = q_i^s(\tilde{\theta}) \) for all \( \theta \in [\tilde{\theta}, \bar{\theta}] \). This means that \( K^s(\theta) = \frac{4}{11} H(z(\tilde{\theta})) + \theta = K^s(\tilde{\theta}) \) for all \( \theta \in [\tilde{\theta}, \bar{\theta}] \). To prove the last part of the Proposition, observe first that (a) \( K^s(\theta) - K^f(\theta) = \frac{4}{11}(H(z(\theta)) - H(\theta)) > 0 \) for all \( \theta \in (\tilde{\theta}, \bar{\theta}) \). Observe next that (b) for all \( \theta \in (\tilde{\theta}, \bar{\theta}) \), \( K^s(\theta) = K^s(\tilde{\theta}) \) and \( K^f(\theta) \) is increasing at an increasing rate. Lastly, observe that (c) \( K^f(\tilde{\theta}) = \frac{4}{11}\tilde{\theta} + \frac{3}{11}\tilde{\theta} < K^s(\tilde{\theta}) = \frac{4}{11}\tilde{\theta} + \frac{3}{11}\tilde{\theta} \). From observations (a), (b) and (c) it follows that \( K^f(\theta) < K^s(\theta) \) for all \( \theta \in (\tilde{\theta}, \bar{\theta}) \). 

What follows from Proposition 4.6 is that \( K^f(\theta) < K^s(\theta) \) for all \( \theta \in (\tilde{\theta}, \bar{\theta}) \), that is, there is more entry under \( M^s(\theta) \) in comparison to \( M^f(\theta) \). The reason is that the regulator allows entry by considering the gap between the incumbent’s virtual marginal cost and the entrant’s true marginal cost while under complete information, the regulator allows entry by considering the gap between the incumbent’s true (and not virtual) marginal cost and the entrant’s true marginal cost. Since the virtual marginal cost of the incumbent is larger than its true marginal cost, there is more entry under asymmetric information. There is more entry under incomplete information due to the asymmetric regulatory treatment between the incumbent and the entrant. For efficiency reasons, the regulator extracts the private information of the incumbent through a fully or partially separating regulatory mechanism. But due to asymmetric information the incumbent receives information rent. While given the open-access nature of the infrastructure, the entrant’s regulatory regime (the entry conditions) does not depend on its private information. Hence, the regulator pays information rents only to the incumbent which imply that the incumbent’s marginal cost rises from \( \theta \) to \( z(\theta) \). Hence, the regulator allows more entry under asymmetric information simply because production of the incumbent is relatively more costly than that of the entrant.\(^{14}\)

Under \( M^s(\theta) \) two types of inefficient entry is possible. They are: (1) not allowing a potential entrant, more efficient than the incumbent, to operate on the downstream market and (2) allowing a potential entrant, less efficient than the incumbent, to operate on the downstream market. The first type of inefficiency (that is, not allowing a potential entrant, more efficient than the incumbent, to operate on the downstream market) follows from Proposition 4.6. Note that \( K^s(\theta) = \theta \) and \( K^s(\bar{\theta}) < \bar{\theta} \) \( (K^s(\tilde{\theta}) < \bar{\theta}) \) under a fully (partially) separating mechanism guarantees that there are intervals in the domain of the continuously differentiable \( K^s(\theta) \) function such that \( K^s(\theta) < \theta \). However, the other type of inefficiency (that is, allowing a potential entrant, less efficient than the incumbent, to operate on the downstream market) does not follow from Proposition 4.6. While the first type of inefficiency is always true. The second type of inefficiency depends on distribution of the marginal cost of the incumbent. It is easy to see that if, for

\(^{14}\)McAfee and McMillan (1987) have a similar result in the context of auctions.
example, \( f(\cdot) \) follows uniform distribution, that is, if \( f(\theta) = g(\theta) = \frac{1}{\Delta} \) for all \( \theta \in [\bar{\theta}, \underline{\theta}] \), then the second type of inefficiency is also possible. With uniform distribution there will be pooling and in particular the cut-off point is \( \bar{\theta} = \frac{121}{224} \bar{\theta} + \frac{103}{224} \underline{\theta} \). Given that \( K^*(\bar{\theta}) = \frac{11}{14} \bar{\theta} + \frac{3}{14} \underline{\theta} \), it follows that \( K^*(\bar{\theta}) - \bar{\theta} = \frac{55}{224} \Delta > 0 \). Hence, if the marginal cost of the incumbent follows uniform distribution, then \( K^*(\bar{\theta}) > \bar{\theta} \) which implies that there are stretches of the \( K^*(\theta) \) function where the second type of inefficiency arises that is there are intervals in the domain of \( K^*(\theta) \) where \( K^*(\theta) > \theta \). Figure 2 illustrates the entry levels for the uniform distribution.

![Figure 2: The entry levels under complete and incomplete information for \( f(\theta) = \frac{1}{\Delta} \).](image)

5 Conclusions

The main conclusion of this paper is that granting non-discriminatory access of the essential facility to a competitor is welfare improving but it generates inefficient entry. Welfare improvement is due to (1) the contribution of the entrant to infrastructure financing and (2) lower information rent paid to the incumbent when the regulator is unaware of the incumbent’s marginal cost. Competition leads to a larger consumer surplus only when there is bunching of less efficient incumbents. In the other cases, the expected price is the same as in the regulated monopoly regime and competition allows the regulator to reduce her transfer. As a result, we have a rise in social welfare.
The regulatory mechanism takes care of the conflict between efficient market structure and infrastructure financing. Under known cost of the incumbent, entry ban of more efficient competitor aims to finance the infrastructure. By allowing only the entry of competitor with a sufficiently large cost advantage, the regulator can extract a larger contribution towards the cost of the essential input. Under unknown cost, the regulator allows the entry of competitor with a sufficiently large cost advantage over the virtual marginal cost. Since the virtual marginal cost is above the marginal cost, there is more entry compared to the known cost case. Moreover, the incumbent’s production decrease with the virtual marginal cost thereby generating more entry. Depending on the distribution of the incumbent’s marginal cost, the other form of inefficiency arises, that is, a less efficient competitor enters the market.

There is more competition on the downstream market under asymmetric information than under symmetric information. This is in sharp contrast with Dana and Spier (1994) and Caillaud and Tirole (2003), where incomplete information reduces competition. In Dana and Spier (1994), where the regulator only regulates the market structure, monopoly production is more likely under asymmetric information. Moreover, the monopoly right is not necessarily granted to the more efficient firm. The market could be operated alone by the high cost firm when the low cost firm has a larger virtual marginal cost than the high cost one. This can happen only if the marginal costs are not distributed according to the same density function. In our framework, this could happen even if \( \theta \) and \( \phi \) are distributed according to the same distribution. But there is no exclusive right: except for the most efficient incumbent, the regulator never grants a monopoly franchise and all types of incumbent face a positive probability of entry.

In Caillaud (1990), the incumbent shuts down whenever the virtual marginal cost of the incumbent is larger than the expected marginal cost of the entrants \( (z(\theta) > E(\phi)) \), provided that the marginal costs of the incumbent and the competitive fringe are distributed independently. Hence, the mechanism allows the entry of less efficient competitors, when on average the entrants’ technology is more efficient than the incumbent’s one. Being regulated, the cost of the incumbent’s technology includes an information rent. When the incumbent operates, the price is maintained at the level of the regulated monopoly regime, but the incumbent produces less. Given that the regulator cannot deter entry by setting a large fixed access charge (since the entrants bypass the incumbent’s input), there is more entry compared to our setting.

In a regulated market, open to competition, an efficient market structure is not granted since market efficiency conflicts with other objectives of the regulator, like infrastructure financing.
6 Appendix

PROOF OF PROPOSITION 4.3: From condition (4.1) it follows that in stage 2, the optimal quantity of the entrant is

\[ q_e(V) = \begin{cases} 
0 & \text{if } \phi \geq K(\theta) \\
\frac{a - \phi - \alpha(\theta) - b q_e(V(\theta))}{2b} & \text{if } \phi < K(\theta) 
\end{cases} \]

where the entry limit is \( K(\theta) = P(q_e(\theta)) - \alpha(\theta) - 2\sqrt{bA(\theta)} \) and zero quantity refers to the no entry case. In stage 1, the regulator incorporates this entry decision in her optimization program (I). Using the incumbent’s profit function \( \Pi_i(\theta) \) we substitute

\[ P(Q(\theta))q_i(\theta) + t(\theta) = \begin{cases} 
\Pi_i(\theta) + c + \theta q_i(\theta) & \text{if no entry} \\
\Pi_i(\theta) + c + \theta q_i(\theta) - \alpha(\theta)q_e(V(\theta)) - A(\theta) & \text{if entry} 
\end{cases} \]

in (I). This substitution means that the regulator’s objective function for any type \( \theta \in \theta \] is

\[ W_i(\theta) = \frac{1}{\Delta} \left[ \int_{\phi=\hat{\Pi}_i(\theta)}^{\phi=K(\theta)} \{ S(Q(\theta)) - (P(Q(\theta)) - \alpha(\theta))q_e(V(\theta)) + A(\theta) \} d\phi + \int_{\phi=K(\theta)}^{\phi=\hat{\Pi}_i(\theta)} S(q_i(\theta))d\phi \right] - \theta q_i(\theta) - c - \hat{\Pi}_i(\theta) \]

Simplifying the function \( W_i(\theta) \), by substituting \( S(Q(\theta)) = \int_0^{Q_i(\theta)} P(x)dx + \int_{Q_i(\theta)}^{Q(\theta)} P(x)dx \) and by substituting \( Q(\theta) = q_i(\theta) + q_e(V(\theta)) \) in the first integrand, we get

\[ W_i(\theta) = (a - \theta)q_i(\theta) - \frac{b(q_i(\theta))^2}{2} + \frac{A(\theta)J_i(\theta)}{\Delta} - \frac{7\sqrt{bA(\theta)}}{3\Delta} - \frac{\alpha(\theta)A(\theta)}{\Delta} + \frac{\alpha(\theta)J_i^2(\theta)}{4b\Delta} + \frac{J_i^3(\theta)}{24b\Delta} - c \] (6.4)

where \( J_i(\theta) \equiv P(q_i(\theta)) - \theta - \alpha(\theta) \). Given the objective function (6.4), it is obvious that, since the regulator wants to ensure the participation constraint of the incumbent and since the cost of the incumbent is common knowledge, she will select \( q_i(\theta) \), \( A(\theta) \), \( \alpha(\theta) \) and \( t(\theta) \) in such a way that \( \hat{\Pi}_i(\theta) = 0 \). Incorporating \( \hat{\Pi}_i(\theta) = 0 \) in (6.4) we get

\[ W^f(\theta) = (a - \theta)q_i(\theta) - \frac{b(q_i(\theta))^2}{2} + \frac{A(\theta)J_i(\theta)}{\Delta} - \frac{7\sqrt{bA(\theta)}}{3\Delta} - \frac{\alpha(\theta)A(\theta)}{\Delta} + \frac{\alpha(\theta)J_i^2(\theta)}{4b\Delta} + \frac{J_i^3(\theta)}{24b\Delta} - c \] (6.5)

Therefore, the problem of the regulator now reduces to the selection of \( q_i(\theta) \), \( A(\theta) \) and \( \alpha(\theta) \) that maximizes \( W^f(\theta) \). The partial derivatives of \( W^f(\theta) \) with respect to \( q_i(\theta) \), \( A(\theta) \) and \( \alpha(\theta) \) are

\[ \frac{\partial W^f(\theta)}{\partial q_i(\theta)} = P(q_i(\theta)) - \theta - \frac{A(\theta)b}{\Delta} - \frac{\alpha(\theta)J_i(\theta)}{2\Delta} - \frac{J_i^2(\theta)}{8\Delta} \] (6.6)

\[ \frac{\partial W^f(\theta)}{\partial A(\theta)} = \frac{J_i(\theta)}{\Delta} - \frac{7\sqrt{bA(\theta)}}{2\Delta} - \frac{\alpha(\theta)}{\Delta} \] (6.7)
\[
\frac{\partial W^f(\theta)}{\partial \alpha(\theta)} = -\frac{2A(\theta)}{\Delta} + \frac{J^2(\theta)}{8\Delta} - \frac{\alpha(\theta)J(\theta)}{2b\Delta}
\]

(6.8)

respectively.

From (6.7) we get \( A(\theta) = \frac{4}{49}(J(\theta) - \alpha(\theta))^2 \) or

\[
49bA(\theta) - 4J^2(\theta) + 8J(\theta)\alpha(\theta) = 4\alpha^2(\theta)
\]

(6.9)

From (6.8) we get

\[
16bA(\theta) - J^2(\theta) + 4J(\theta)\alpha(\theta) = 0
\]

(6.10)

From (6.9) and (6.10) we get

\[
15J^2(\theta) + 68J(\theta)\alpha(\theta) + 64\alpha^2(\theta) = 0
\]

(6.11)

Solving (6.11) we get

\[
\alpha(\theta) = -3(P(q_i(\theta)) - \theta)
\]

(6.12)

and

\[
\alpha(\theta) = -\frac{5}{11}(P(q_i(\theta)) - \theta)
\]

(6.13)

If (6.12) holds, then from (6.9) we get \( A(\theta) = \frac{4}{49}\alpha^2(\theta) \). By substituting \( A(\theta) = \frac{4}{49}\alpha^2(\theta) \) in (6.6), we have after simplifications: \( P(q_i(\theta)) = \theta \). Therefore, one possible candidate solution is \( V_1(\theta) = (q_i(\theta) = \frac{\alpha - \theta}{\theta}, A(\theta) = \frac{4}{25}(\theta - \theta)^2, \alpha(\theta) = -3(\theta - \theta)) \). This solution implies a total entry ban: \( K(\theta) = \theta \forall \theta \). If (6.13) holds, then from (6.9) we get \( A(\theta) = \frac{36}{25}\alpha^2(\theta) \). By substituting \( A(\theta) = \frac{36}{25}\alpha^2(\theta) \) in (6.6), we have after simplifications:

\[
P(q_i(\theta)) = \theta + \frac{28}{25}\frac{\alpha^2(\theta)}{\Delta}
\]

(6.14)

From (6.13) and (6.14), we get

\[
-\frac{11}{5}\alpha(\theta) + \theta = \theta + \frac{28}{25}\frac{\alpha^2(\theta)}{\Delta}
\]

(6.15)

Solving for \( \alpha(\theta) \), we get

\[
\alpha(\theta) = -\frac{5}{56}(11\Delta \pm \sqrt{121\Delta^2 - 112\Delta(\theta - \theta})
\]

(6.16)
Since at $\theta$, $\alpha(\theta) = 0$, we get $\alpha(\theta) = -\frac{5}{36}(11\Delta - \sqrt{121\Delta^2 - 112\Delta(\theta - \bar{\theta})})$. Then from condition (6.13), we get $P(q_i(\theta)) = \theta - \frac{11}{36}\theta$. Thus, the other possible candidate solution is $V_2(\theta) = \langle q_i(\theta) = \frac{\alpha - H(\theta)}{b}, \alpha(\theta) = -\frac{5}{11}(H(\theta) - \bar{\theta}), A(\theta) = \frac{36(H(\theta) - \bar{\theta})^2}{121b} \rangle$ where $H(\theta) = \theta + 112\Delta - 11\sqrt{121\Delta^2 - 112\Delta(\theta - \bar{\theta})}$. From the expression of $H(\theta)$, it is easy to check that $H(\theta) = \theta$ and $H(\theta) > \theta$ for all $\theta \in (\bar{\theta}, \bar{\theta})$. Differentiating $H(\theta)$ twice with respect to $\theta$ we get that $H(\theta)$ is strictly increasing and strictly convex in $\theta \in (\bar{\theta}, \bar{\theta})$. With solution $V_2(\theta)$, there is an entry possibility. In particular, the entry limit is $K(\theta) = \theta + \frac{11}{12}[121\Delta - 11\sqrt{121\Delta^2 - 112\Delta(\theta - \bar{\theta})}]$. Here $K(\theta) \in (\bar{\theta}, \theta)$ for all $\theta \in (\bar{\theta}, \bar{\theta})$.

It is straightforward to show that the welfare associated with the solution $V_2(\theta)$ is larger than the no entry solution $V_1(\theta)$ for all $\theta \in (\bar{\theta}, \bar{\theta})$. For any type $\theta \in [\bar{\theta}, \bar{\theta}]$, the welfare difference between the entry and no entry case is

$$W_f(\theta)|_{V_2(\theta)} - W_f(\theta)|_{V_1(\theta)} = \frac{1}{36} \left[ 1 - \frac{11}{12} \sqrt{1 - \frac{1 - 121\Delta(\theta - \bar{\theta})}{121\Delta}} \right] \right] (6.17)$$

Condition (6.17) is obtained after substituting the values of $V_2(\theta)$ and $V_1(\theta)$ in $W_f(\theta)|_{V_2(\theta)}$ and $W_f(\theta)|_{V_1(\theta)}$ respectively and then simplifying $W_f(\theta)|_{V_2(\theta)} - W_f(\theta)|_{V_1(\theta)}$ by using the condition

$$(H(\theta) - \bar{\theta})^2 = \frac{121\Delta}{28}(H(\theta) - \theta).$$

From condition (6.17) it follows that $W_f(\theta)|_{V_2(\theta)} = W_f(\theta)|_{V_1(\theta)}$ since $H(\theta) = \theta$. More importantly, from condition (6.17) it also follows that $W_f(\theta)|_{V_2(\theta)} > W_f(\theta)|_{V_1(\theta)}$ for all $\theta \in (\bar{\theta}, \bar{\theta})$ since the right hand side of condition (6.17) is then strictly positive. Hence, solution $V_2(\theta)$ gives a higher welfare than the no entry solution $V_1(\theta)$.

To verify the second order condition under $V_2(\theta)$, we first incorporate the values of $\alpha(\theta) = -\frac{5}{11}(P(q_i(\theta)) - \theta)$ and $A(\theta) = \frac{36(P(q_i(\theta)) - \theta)^2}{121b}$ as a function of the quantity $q_i(\theta)$ in $W_f(\theta)$. This gives

$$W_f(\theta) = (a - \theta)q_i(\theta) - \frac{b(q_i(\theta))^2}{2} + \frac{28(P(q_i(\theta)) - \theta)^3}{363b\Delta} - c \quad (6.18)$$

The second derivative of $W_f(\theta)$ with respect to $q_i(\theta)$ gives

$$\frac{\partial^2 W_f(\theta)}{\partial q_i(\theta)^2} = -b\sqrt{121\Delta^2 - 112\Delta(\theta - \bar{\theta})} \quad (6.19)$$

Clearly, $\frac{\partial^2 W_f(\theta)}{\partial q_i(\theta)^2} < 0$ for all $\theta \in (\bar{\theta}, \bar{\theta})$. Hence the optimal solution to the regulator’s problem is $V_2(\theta) = \langle q_i^i(\theta) = \frac{a - H(\theta)}{b}, A^i(\theta) = \frac{36(H(\theta) - \theta)^2}{121b} \rangle, \alpha^i(\theta) = -\frac{5}{11}(H(\theta) - \theta) \rangle$ where $H(\theta) = \theta + 112\Delta - 11\sqrt{121\Delta^2 - 112\Delta(\theta - \bar{\theta})}$. The optimal transfer is obtained from the profit expression $\hat{\Pi}_i^i(\theta) = 0$. Recall that $\hat{\Pi}_i^i(\theta) = 0$ implies that

$$\frac{1}{\Delta} \left[ \int_{\phi=\bar{\theta}}^{\phi=\bar{\theta}} \left\{ P(Q^i(\theta))q_i^i(\theta) + \alpha^i(\theta)q_i(\phi, V_2(\theta), \phi) + A^i(\theta) \right\} d\phi + \int_{\phi=\bar{\theta}}^{\phi=\bar{\theta}} P(q_i^i(\theta)q_i^i(\theta) d\phi \right] - c - \theta q_i(\theta) + T^i(\theta) = 0 \quad (6.20)$$

15 This condition follows quite easily by simplifying $(H(\theta) - \theta)^2$. 20
Using the optimal solution \( V_2(\theta) \), it is easy to verify that expected market price for any \( \theta \in [\underline{\theta}, \overline{\theta}] \) is

\[
\hat{P}^I(\theta) = \left[ \int_{\phi=\theta}^{\phi=K(\theta)} P(Q^I(\theta))d\phi + \int_{\phi=K(\theta)}^{\phi=\overline{\theta}} P(q^I_i(\theta))d\phi \right] = \theta \tag{6.21}
\]

Condition (6.21) is obtained after simplifying \( \hat{P}^I(\theta) \) by using \( \langle H(\theta) - \theta \rangle^2 = \frac{121\Delta}{28}(H(\theta) - \theta) \). By substituting (6.21) in (6.20) and then simplifying it we get

\[
t^I(\theta) = c - \frac{1}{\Delta} \left[ \int_{\phi=\theta}^{\phi=K(\theta)} \{ \alpha(\theta)q_c(V(\theta)) + A(\theta) \} d\phi \right] \tag{6.22}
\]

for all \( \theta \in [\underline{\theta}, \overline{\theta}] \). Finally, one can easily verify that the expected revenue of the incumbent, that is, \( \frac{1}{\Delta} \int_{\phi=\theta}^{\phi=K(\theta)} \{ \alpha(\theta)q_c(V(\theta)) + A(\theta) \} d\phi \) is equal to \( \frac{4(\langle H(\theta) - \theta \rangle^3)}{114\Delta} > 0 \) for all \( \theta \in (\underline{\theta}, \overline{\theta}) \) which implies that \( t^I(\theta) < c \) for all \( \theta \in (\underline{\theta}, \overline{\theta}) \).

**PROOF OF PROPOSITION 4.5:** The objective of the regulator is to select a quadruple \( M = \langle q_i(\theta), \alpha(\theta), A(\theta), t(\theta) \rangle \) that maximizes the objective function \( \hat{W} \equiv \int_{\theta=\underline{\theta}}^{\theta=\overline{\theta}} W(\theta)d\theta \) subject to the incentive compatibility constraint \( \hat{\Pi}_i(\theta) \geq \hat{\Pi}_i(\theta'; \theta), \forall(\theta, \theta') \in [\underline{\theta}, \overline{\theta}]^2 \) and the participation constraint \( \hat{\Pi}_i(\theta) \geq 0, \forall \theta \in [\underline{\theta}, \overline{\theta}] \). Here the social welfare function, when the incumbent’s type is \( \theta \), is given by \( W(\theta) = \frac{1}{\Delta} \int_{\phi=\theta}^{\phi=K(\theta)} \{ S(Q(\theta)) - P(Q(\theta))Q(\theta) - t(\theta) \} d\phi \). Following the same steps as in the proof of Proposition 4.3 (for simplifying \( W(\theta) \)), one can rewrite the problem of the regulator as one of maximizing \( \hat{W} \equiv \int_{\theta=\underline{\theta}}^{\theta=\overline{\theta}} W_1(\theta)d\theta \) subject to (i) \( \hat{\Pi}_i(\theta) \geq \hat{\Pi}_i(\theta'; \theta), \forall(\theta, \theta') \in [\underline{\theta}, \overline{\theta}]^2 \) and (ii) \( \hat{\Pi}_i(\theta) \geq 0, \forall \theta \in [\underline{\theta}, \overline{\theta}] \) where \( W_1(\theta) \) is given by (6.4). From the incentive constraint and participation constraint we know that \( \hat{\Pi}_i(\theta) = \int_{\tau=0}^{\tau=\overline{\theta}} q_i(\tau)d\tau \). Incorporating this restriction on expected profit of the incumbent, we can write the objective function of the regulator as \( \hat{W} = \int_{\theta=\underline{\theta}}^{\theta=\overline{\theta}} W_1(\theta)d\theta \) where

\[
W_1(\theta) = a - \theta q_i(\theta) - \frac{b(q_i(\theta))^2}{2} + A(\theta)J(\theta) - 7\sqrt{5}(A(\theta))^2 \cdot \frac{\sqrt{\Delta}}{3\Delta} - \frac{\alpha(\theta)A(\theta)}{4\Delta} + \frac{\alpha(\theta)J^2(\theta)}{24\Delta} + \frac{J^3(\theta)}{24\Delta} - c - \int_{0}^{\overline{\theta}} q^i(\tau)d\tau \tag{6.23}
\]

Since integrating \( \int_{\theta=\underline{\theta}}^{\theta=\overline{\theta}} \{ \int_{\tau=0}^{\tau=\overline{\theta}} q_i(\tau)d\tau \} d\theta \) by parts we get \( \int_{\theta=\underline{\theta}}^{\theta=\overline{\theta}} \{ \int_{\tau=0}^{\tau=\overline{\theta}} q_i(\tau)d\tau \} d\theta = \int_{\theta=\underline{\theta}}^{\theta=\overline{\theta}} L(\theta)q_i(\theta)d\theta \), we can finally write down the objective function of the regulator as

\[
\hat{W} = \int_{\theta=\underline{\theta}}^{\theta=\overline{\theta}} W_1^*(\theta)d\theta \tag{6.24}
\]

The optimization can now be done pointwise, that is, the regulator’s problem now is to select \( q_i(\theta), A(\theta) \) and \( \alpha(\theta) \) that maximizes \( W_1^*(\theta) \) (given by (6.24)) for each \( \theta \). This problem is similar to that of maximizing the problem in the proof of Proposition (4.3) where the regulator’s problem was to select \( q_i(\theta), A(\theta) \) and \( \alpha(\theta) \) to maximize \( W_1^I(\theta) \) given by (6.5). The only difference between \( W_1^I(\theta) \) and \( W_1^*(\theta) \) is that while in the former we had a term \( -\theta q_i(\theta) \), in the latter this term is replaced by \(-z(\theta)q_i(\theta)\). Thus, following similar steps we get the optimal quantity of the incumbent and the optimal access charges as \( q_i^*(\theta) = q_i^I(z(\theta)), A^*(\theta) = A^I(z(\theta)) \) and \( \alpha^*(\theta) = \alpha^I(z(\theta)) \).
respectively for each $\theta \in [\underline{\theta}, \bar{\theta}]$. Thus, the optimal solution is $V^*(\theta) = \langle q_i^*(\theta), A^*(\theta), \alpha^*(\theta) \rangle$. It is important to note that like in the known cost case the entry ban solution under asymmetric information (that is, $V(\theta) = \langle q_i(\theta) = \frac{a-z(\theta)}{b}, A(\theta) = \frac{4}{b}(z(\theta) - \bar{\theta})^2, \alpha(\theta) = -3(z(\theta) - \bar{\theta}) \rangle$) is pointwise sub-optimal since, for each type $\theta$, it yields a welfare not higher than the optimal solution $V^*(\theta)$. Finally, by substituting the optimal $q_i^*(\theta)$, $A^*(\theta)$ and $\alpha^*(\theta)$ in (4.3) and setting $\hat{P}_i(\theta) = \int_0^{\bar{\theta}} q_i(\tau) d\tau$, we get $t^*(\theta) = t^f(z(\theta)) - L(\theta)q_i^*(\theta) + \int_0^{\bar{\theta}} q_i^*(\tau) d\tau$.

If the distribution $f(\theta)$ is such that $z(\bar{\theta}) \leq \theta + \frac{9}{112} \Delta$, then $H(z(\theta))$ is defined for all $\theta \in [\underline{\theta}, \bar{\theta}]$ and the solution given above is valid. If, however, $z(\theta) > \theta + \frac{9}{112} \Delta$ then we have an existence problem. This means that $H(z(\theta)) = \theta + \frac{121 \Delta - 11 \sqrt{121 \Delta^2 - 112 \Delta (z(\theta) - \bar{\theta})}}{56}$ is not defined when $z(\theta)$ is above $\theta + \frac{9}{112} \Delta$. Given that $z(\theta)$ is increasing in $\theta$, there is an existence problem for all $\theta \in [\underline{\theta}, \bar{\theta}]$ where $\bar{\theta} = z^{-1}(\theta + \frac{9}{112} \Delta)$. The existence problem comes from the non concavity of the $W_1^*(\theta)$ function for all $\theta$ such that $z(\theta) > \theta + \frac{9}{112} \Delta$. When we incorporate the optimal values of $\alpha^*(\theta)$, $A^*(\theta)$ in $W_1^*(\theta)$, we get:

$$W_1^*(\theta) = (a - z(\theta))q_i(\theta) - \frac{b q_i(\theta)^2}{2} + \frac{28(P(q_i(\theta)) - \bar{\theta})^3}{3636\Delta} - c \quad (6.25)$$

The second derivative of $W_1^*(\theta)$ with respect to $q_i^*(\theta)$ gives:

$$\frac{\partial^2 W_1^*(\theta)}{\partial q_i^*(\theta)^2} = -b \frac{\sqrt{121 \Delta^2 - 112 \Delta (z(\theta) - \bar{\theta})}}{11 \Delta} \quad (6.26)$$

The second derivative in condition (6.26) is negative only if $z(\theta) < \theta + \frac{9}{112} \Delta$ and not defined otherwise. Hence, if the distribution $f(\theta)$ is such that (6.26) is not defined, the above solution is not valid for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

To solve this non-existence problem with pointwise optimization we incorporate optimum bunching procedure. For that we need to apply optimal control theory. We break the regulator’s problem into two sub-problems. We define an interval $[\underline{\hat{\theta}}, \bar{\theta}]$ where we have full separability of types and the interval $[\underline{\theta}, \bar{\theta}]$ where we have pooling of types and $\hat{\theta}$ is an, as yet undetermined, cut-off point. The incentive compatibility problem in terms of the first derivative (that is, $\frac{\partial \hat{P}_i(\theta)}{\partial \theta} = -q_i(\theta)$) acts as the equation of motion in the two sub-problems and the condition $\hat{P}_i(\bar{\theta}) = 0$ (obtained from participation constraint) is the transversality condition. We solve for the two sub-problems and finally select the optimal cut-off point $\hat{\theta}$. This cut-off point turns out to be a point where we had existence problem with pointwise maximization. Thus, the optimal solution is identical to the pointwise optimization problem in the well-defined zone and is a pooling one for higher types. Finally, it is quite easy to verify that this solution weakly dominates the monopoly solution $V(\theta) = \langle q_i(\theta) = \frac{a-z(\theta)}{b}, A(\theta) = \frac{4}{b}(z(\theta) - \bar{\theta})^2, \alpha(\theta) = -3(z(\theta) - \bar{\theta}) \rangle$ pointwise.

\[\blacksquare\]
References


