IMPORT RESTRAINTS AND HORIZONTAL PRODUCT DIFFERENTIATION

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ABSTRACT

We consider the impact of an import quota under price competition in the Hotelling model of horizontal product differentiation. Two issues are contemplated. First, we show that the main qualitative implication of the quota in a pricing game is to generate equilibrium outcomes quite similar to those prevailing under Cournot competition. In particular the optimal quota from the domestic point of view is invariant to the mode of competition. Second, we show how the presence of the quota affects the choice of products' attributes. When transportation costs are quadratic, the maximum differentiation principle does not hold for most values of the quota: by relaxing price competition, the quota reverses firms' incentives with respect to the choice of attributes.

Keywords: Hotelling, Optimal Quota, Price Competition

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1) Introduction

The relative effects of tariffs and quotas is a traditional concern in the theory of international trade policy. It is well known indeed that tariffs need not be equivalent to quotas under imperfect competition. A seminal contribution in this respect is Krishna [89]. She shows that a quota imposed at the free trade equilibrium (hereafter FTE) level dramatically affects market outcomes under price competition, whereas it is completely ineffective under quantity competition. The contribution of Krishna is twofold. First, she shows that the qualitative implications of a quota go beyond those of tariffs because the specific implication of the quota is to alter directly firms’ incentives over the whole competition game. This latter point explains why apparently innocent quotas do affect equilibrium outcomes. Second, because price competition under quotas is formally equivalent to price competition game with one firm facing a capacity constraint, she offers an original equilibrium characterisation in a class of Bertrand-Edgeworth games with product differentiation.

As reported by Brander [95], in the field of international economics, few papers have elaborated on the preliminary results of Krishna. Most papers dealing with quotas restrict attention to quantity and/or quality setting games. The precise implications of quotas under price competition have comparatively received much less attention in the literature. Moreover, recent empirical work dealing with price setting and quotas in international markets often bypasses the issue raised by Krishna. Indeed the assumed existence of a price equilibrium in pure strategies is at odds with the theoretical results of Krishna who basically shows that a quota typically destroys such pure strategy equilibria. It is fair to say however that very little is known about the exact dependence of the effects emphasised in Krishna’s paper on the degree of product differentiation. Moreover, her analysis concentrates on quotas in the vicinity of free trade. From an industrial organisation perspective, Krishna’s contribution deserves to be emphasised also. While there exists a vast literature dealing with Bertrand-Edgeworth competition when products are homogeneous, very few papers explicitly addressed the issue under product differentiation. More theory seems thus to be called for in the field of price competition in the presence of quotas.

In the present paper we consider two related issues. First, we highlight the implication of the quota for the nature of competition. In this respect, we show that the chief attribute of the quota is to turn price competition into quantity-like competition. This result is noteworthy because strategic trade policy recommendations are often heavily dependent on the mode of competition. In this respect, we suggest that the quota may reconcile equilibrium outcomes resulting from price competition with those of quantity competition. In our particular model, this intuition is strikingly confirmed: the level of the optimal quota is indeed totally invariant to the mode of competition.

Second, we study the choice of products’ characteristics in the presence of an import quota. Given its impact on price competition we clearly expect the quota to affect the choice of products’ attributes. Similar issues have been widely studied under Cournot competition and for vertical differentiation. Das and Donnenfeld [89], Herguera, Kujal & Petrakis [99] and our companion paper Boccard & Wauthy [98] are

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1See for instance Verboven [96], Goldberg [95], Goldberg & Verboven [98]. Feenstra [87] assumes that firms are price-takers.  
2Furth and Kovenock [93] counts among the rare exceptions we are aware of.
recent examples. We focus here on horizontal differentiation and show that firms' incentives to differentiate horizontally are basically reversed as compared to free trade result of maximal differentiation.

Our results are also noteworthy as preliminary contributions to the analysis of capacity-constrained pricing games in differentiated markets. In our model, the presence of a quantitative restriction (the quota) is shown to turn price competition into quantity-like competition: equilibrium payoffs are quite similar to those prevailing should competition take place in a Cournot fashion. Moreover, the foreign firm would voluntarily constrain herself. This is highly reminiscent of the analysis conducted by Kreps and Scheinkman [83] in a market for homogeneous goods. On the other hand, our analysis reveals the presence of quantitative constraints is not likely to foster minimal differentiation under horizontal differentiation, contrarily to what happens under vertical differentiation (Boccard and Wauthy [98]).

In order to establish our results, we model horizontal product differentiation using the Hotelling [29] framework. As noted by Krishna [89] page 260, this framework offers a natural application of her analysis. Furthermore, it has been used recently in the literature on international trade (see for instance Schmitt [90], [95]). Indeed, it neatly captures the idea that intra-industry trade occurs because of the variety in consumers' tastes and products' characteristics while allowing for strategic interactions between firms. This model offers a convenient framework to study in full details the implications of a quota under horizontal differentiation which is the main goal of the present paper. To this end, we consider a game similar to that of Krishna [89] where a domestic and a foreign producer compete in prices on the domestic market. The foreign producer is facing a quota when price competition takes place. Products are imperfect substitutes. We first characterise Nash equilibrium in prices for all possible values of the quota (section 2). Section 3 is devoted to the analysis of the optimal level for the quota. In section 4, we characterise Cournot equilibria and establish our invariance result. Section 5 deals with the choice of products' attributes. Section 6 concludes.

2) PRICE EQUILIBRIUM IN THE PRESENCE OF A QUOTA

In this section, we characterise the Nash equilibrium in prices. We first analyse a simplified Hotelling model under free trade as it provides a useful starting point for the (more involved) analysis of price competition with the quota.

2.1) THE FREE TRADE BENCHMARK

Consider a domestic market consisting of a street of unit length. An homogeneous good is sold at two shops. One sells a domestic product at a price $p_d$ and the other one sells a foreign product at a price $p_f$. They are respectively located at the left end and the right end of the unit segment along which consumers are uniformly distributed. Each consumer is identified by its address $x \in [0,1]$ in the street. An agent buys at most one unit of the good, the common reservation price is $S$. When buying one of the products, the consumer goes to a shop and bears a transportation cost linear in the distance to the shop. Since we can
normalise prices, we set the transportation cost between the two shops at 1$. Therefore, our parameter $S$ measures the indirect utility of consuming one unit of the differentiated good, deflated by the unit transportation cost. Hence, a large $S$ either means that consumers like the good very much or that the differentiation dimension is relatively unimportant in the total valuation of the good.

The utility derived by a consumer located at $x$ in the interval $[0,1]$ is

$$\begin{align*}
S - (1 - x) - p_d & \text{ if the product is bought at the foreign firm} \\
S - x - p_f & \text{ if the product is bought at the domestic firm}
\end{align*}$$

Refraining from consuming any of the two products yields a nil level of utility\(^4\). We characterise the Hotelling equilibrium in the following Lemma.

**Lemma 1 (Hotelling)**

*If $S > 3/2$ and firms face no quantitative constraints, the only Nash equilibrium of the pricing game is $(1,1)$ and the market is covered.*

**Proof** If prices are low all agents buy the good (the market is covered). The indifferent agent is located at $\tilde{x}(p_d,p_f) \equiv \frac{1 - p_d + p_f}{2}$, the solution of $S - x - p_d = S - (1 - x) - p_f$. The domestic firm receives demand $\tilde{x}(p_d,p_f)$ and the foreign firm $1 - \tilde{x}(p_d,p_f)$. The market is not covered if prices are high. In that case, firm\(^5\) $i$ is a local monopoly and its demand is $\min\{1,S - p_i\}$. This happens if $S - \tilde{x}(p_i,p_j) - p_i < 0 \iff p_i > 2S - 1 - p_j$. The demand addressed to firm $i$ is $D_i(p_i,p_j) = \begin{cases} 
\frac{1 - p_i + p_j}{2} & \text{if } p_i \leq 2S - 1 - p_j \\
\min\{1,S - p_i\} & \text{if } p_i > 2S - 1 - p_j
\end{cases}$. The maximisers are $H(p_j) \equiv \frac{1 + p_j}{2}$ and the monopoly price $p^m \equiv \min\{S - 1,S/2\}$. As $D_i(p_i,p_j)$ is piecewise linear and decreasing in $p_j$, the profit function is concave in $p_j$. Thus, the best reply to a mixed strategy is the best reply to its expectation, which is a pure strategy. Therefore, the unique Nash equilibrium of this game is pure. The best reply of the domestic firm, $BR_d$ is equal to $H(p_f)$ for $p_f$ less than $4S/3 - 1$. Then firms are not competing anymore and the best reply is to adjust the price at $2S - 1 - p_j$ to cover the market. This conduct last until the monopoly price is reached. The best reply intersect at the unit price for both firms as soon as $S > 3/2$. Otherwise, there is a continuum of equilibria on the frontier which entail no "real" price competition.◆

We shall assume $S > 3/2$ in the sequel of the paper, i.e. we focus on parameters' constellations such that firms effectively compete under free trade. Note that the equilibrium prices do not depend on $S$ and in equilibrium all consumers enjoy a strictly positive surplus. Therefore, firms could benefit from the presence of the quota to relax price competition. Indeed, they could realise identical sales by raising their prices simultaneously.

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\(^3\) Although we use the "location" terminology, our analysis is developed in the traditional horizontal differentiation models: the "Street" denotes the space of products' possible attributes, consumers are distributed according to their tastes and firms' "locations" define specific choice of attributes.

\(^4\) In Hotelling’s original model, this possibility is not considered, formally, this correspond to an infinite $S$.

\(^5\) In the remainder of the text, $i$ stands for either of the firm and $j$ for her competitor.
2.2) Import Restraints

In order to capture the intuition underlying our analysis, let us sketch Krishna [89]’s argument. Consider two firms selling substitute products and competing in prices in a differentiated industry. We assume that the unique Nash equilibrium which prevails when firms do not face any form of quantitative restriction is \((p_1^*, p_2^*)\). Suppose now that firm 2 is facing a quota at the FTE level, i.e. \(q = D_2(p_1^*, p_2^*)\). Is \((p_1^*, p_2^*)\) still an equilibrium? Presumably not. Indeed, if firm 1 raises \(p_1\), against \(p_2^*\), its demand should decrease whereas the demand addressed to firm 2 should increase. However, firm 2 cannot meet this demand as it exceeds the quota. Accordingly, rationing appears in the market. It is then sufficient that some rationed consumers turn back to firm 1 in order to make the deviation profitable, thereby destroying our equilibrium candidate.

The preceding argument is reminiscent of Edgeworth [25]: the domestic producer has an incentive to name high prices in order to create rationing at the foreign firm and benefit from those rationed consumers who turn back to it. The quota allows the domestic firm to act as a monopolist along a residual demand. The level of this residual demand depends on the level of the quota and the extent to which rationed consumers are willing to buy the domestic product instead of refraining from consuming. Other things being equal, the larger the residual demand, the greater the incentives to quote high prices. Therefore, the extent to which the domestic producer recovers rationed consumers is of crucial importance for the analysis. In the Hotelling model with fixed locations, consumers have unit demand for the products and are characterised by a particular reservation price for each product. Therefore the identity of the rationed consumers is directly linked to their willingness to report their purchase to the domestic firm in case of rationing.

We assume, as Kreps & Scheinman [83], that the efficient rationing rule is at work in the market so that consumers nearest to the shops get served first. This rule easily compares with the implicit one considered by Krishna as it amounts to assume that consumers are able to exhaust at no cost any arbitraging possibilities that would prevail in a market with rationing. It follows that ultimately, rationed consumers have the lowest reservation prices for the foreign product. Consider the example depicted on figure 1 below where \(p_d > p_f\) and \(q\) is small. Some consumers willing to buy at the foreign firm are rationed. Under efficient rationing, they are located in the interval \([\tilde{x}(p_d, p_f); 1 - q]\) and thus are precisely the most inclined to switch to the domestic firm. Despite the latter has a potentially low demand, the binding quota on the foreign firm gives the domestic firm an effective demand of \(1 - q\). More precisely, as long as \(p_d\) is less than \(S - (1 - q)\), which measures the net utility of the consumer located in \(q\), the effective demand of the domestic firm is \(1 - q\).

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6 In fact, it is sufficient that the foreign firm is not willing to sell beyond the quota. This is the case whenever selling beyond the quota involves some form of sanction imposed by the domestic government.

7 In fact, pricing games with quotas are formally equivalent to particular capacity-constraints pricing games where one firm only faces a constraint. Such models have been extensively studied since the pioneering work of Edgeworth [25], although almost exclusively in the case of homogeneous products. Levitan & Shubik [72] in particular consider a game of which our present analysis is the extension to a horizontally differentiated market.

8 Note thus that products’ location is not addressed in the present analysis. However, we discuss some implications of our findings on products’ locations in the last section.

9 We will discuss later on the robustness of our results to the introduction of other rationing rules.
This feature of the market allocation rule also lowers domestic firm's incentives to enter a price competition "à la Bertrand" since its demand is locally independent of its own price \( p_d \).

2.3) THE QUOTA-CONSTRAINED EQUILIBRIUM

Technically speaking, the presence of the quota implies that the payoffs of the domestic producer are not quasi-concave, so that the existence of an equilibrium can be problematic. Yet, contrarily to models of homogeneous goods, payoffs are continuous under product differentiation, therefore it is only the existence of an equilibrium in pure strategies which is problematic.

The analysis of the pricing game with the quota \( q \) proceeds as follows. Against a foreigner's price, the domestic producer contemplates two options: by naming a high price, it will make the quota binding, thereby generating rationing and spillovers. By naming a low price, it fights for market shares, exactly as under free trade. We first characterise the shape of demands corresponding to these strategic options and then compute the firms' best replies. The domestic one is discontinuous, reflecting the two strategic options mentioned above, whereas the foreign one is continuous but kinked. With these best replies in hand, we characterise the Nash equilibrium. This equilibrium may involve either pure or mixed strategies depending on the value of the parameters \((q, S)\).

The full derivation of the demand functions can be found in our paper (Boccard & Wauthy [97]) on capacity pre-commitment in the Hotelling model, which is more general on this point. It is sufficient for our present purpose to rely on Figure 2 below to understand how the quota affects the pricing game.

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**Figure 1**

Potential demand for the **domestic** firm

Potential demand for the **foreign** firm

\[ x \]

0  \( \hat{x}(p_f, p_d) \)  1−q  1

**domestic sales**

**foreign sales**

Rationed consumers

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**Figure 2**

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The full derivation of the demand functions can be found in our paper (Boccard & Wauthy [97]) on capacity pre-commitment in the Hotelling model, which is more general on this point. It is sufficient for our present purpose to rely on Figure 2 below to understand how the quota affects the pricing game.
Two problems have to be taken into account in order to analyse the pricing game. First, because of the quota, demand addressed to firms at prevailing prices may differ from their actual sales. Second, under our assumptions, the market is not covered when prices are high whereas it is inelastic at low prices. Combining these two features, we may identify four critical regions in the price space. Let us start from area A where the two prices are low. The market is covered and the foreign firm is not constrained by the quota. If \( p_f \) increases, we leave area A to get into area D as in the free trade analysis: the marginal consumer \( \tilde{x}(p_d, p_f) \) ceases to buy the good and the market is not covered. The complex part is when \( p_d \) increases because the domestic firm expects to benefit from spillovers. Indeed, in area B, the domestic firm recovers all rationed consumers (i.e. sales are equal to \( 1 - q \)) while for larger prices some consumers cease to buy. This is area C where the foreign producer is still constrained by the quota. Formally, the equation of the frontier A/B is the solution of \( 1 - \tilde{x}(p_d, p_f) = q \) and gives \( p_d = p_f + 1 - 2q \), the equation of the frontier A/D is \( p_d = 2S - 1 - p_f \).

We can now derive the best reply functions by considering in turn the optimal responses in the four areas. Consider first the case of the foreign firm. Against a low \( p_d \) in area A, the foreign firm responds in an aggressive manner to gain market shares: the free trade analysis of lemma 1 applies and the best reply is \( \psi_f(p_d) = \frac{1 + p_d}{2} \) whenever it belongs to area A. Indeed, the demand is \( D_f = 1 - \tilde{x}(p_d, \psi_f(p_d)) = \frac{1 + p_d}{4} \), it reaches the quota at \( \tilde{p}_d = 4q - 1 \). If \( p_d \) is larger, the foreign firm sticks to the quota by playing \( p_d + 1 - 2q \) (the frontier A/B). In areas B and C where the quota is binding, the foreign demand is constant, thus the optimal price is the largest possible one which leads us to the frontiers with areas A and D. Note then that the optimal price in \( B \cup C \) is dominated by that of \( A \cup D \). The only (technical) problem is the domain of monopoly demand D. We assume \( p^m = S/2 < S - q \), thus the monopoly profit function is decreasing everywhere in D and the optimal choice is \( S - q \). In conclusion, the best reply of the foreign firm is the continuous function:

\[
\psi_f(p_d) = \begin{cases} 
\frac{(1 + p_d)}{2} & \text{if } p_d \leq \tilde{p}_d \\
 p_d + (1 - 2q) & \text{if } \tilde{p}_d < p_d < S - 1 + q \\
 S - q & \text{if } S - 1 + q \leq p_d 
\end{cases}
\]

Regarding the domestic best reply, area \( C \cup D \) is straightforward to analyse since there is no competition. The monopolistic price \( S/2 \) is the overall maximiser of the profit. If it lies above \( S - 1 + q \), it is a dominant strategy for the domestic firm in equilibrium. This happens when \( q < 1 - S/2 \) i.e., \( S \) must be small and the quota very restrictive (less than 1/4 as \( S > 3/2 \)). Otherwise, the optimal price is \( S - 1 + q \).

The crucial point that drives all our results is the behaviour of the domestic firm in a competitive context i.e., area \( A \cup B \) where the market is covered. It can act in a classical fashion by fighting for market shares with a low price or it can take advantage of the quota by naming a high price in order to create some rationing at the foreign firm and recover rationed consumers. Intuitively, if the foreign firm is aggressive, the price competition generated by the first option drives profits to zero, it is therefore better to hide behind the quota in order to act as a monopolist on a residual demand. However, if the foreign firm becomes less aggressive then it is optimal to revert to an aggressive pricing. Thus, the optimal behaviour of the domestic firm can drastically change, depending on its perception of the foreign firm pricing.

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10 Otherwise \( q > 3/4 \) as \( S > 3/2 \). The best reply of the domestic firm to \( S/2 \) would create competition and drives us back to area A which means that \( S/2 \) do not appear in equilibrium.
The first option corresponds to area A where the free trade analysis applies, the optimal price is \(\frac{1 + p_f}{2}\). The second option is area B where the demand is always \(1 - q\), the largest price \(S - 1 + q\) is therefore optimal. The associated profits are respectively \(\frac{(1 + p_f)^2}{8}\) and \([S - 1 + q](1 - q)\), they are equal at \(\hat{p}_f = \sqrt{8(S - 1 + q)(1 - q)} - 1\).

We obtain the discontinuous best reply function of the domestic firm:

- if \(q < 1 - S/2\), \(\psi_d(p_f) = S/2\)
- if \(q \geq 1 - S/2\), \(\psi_d(p_f) = \begin{cases} S - 1 + q & \text{if } p_f \leq \hat{p}_f \\ \frac{1 + p_f}{2} & \text{if } \hat{p}_f < p_f < 3S/2 - 1 \\ S/2 & \text{if } 3S/2 - 1 \leq p_f \end{cases}\)

The best reply function of the domestic firm being discontinuous at \(\hat{p}_f\) the existence of pure strategy equilibria is problematic. A mixed strategy equilibrium always exists because payoffs are continuous but contrarily to the case studied by Levitan & Shubik [72], there is no density of prices in equilibrium. This is a central property of pricing models of product differentiation under quantitative restrictions, as already shown by Krishna [89]. In the next Proposition, we characterise the Nash equilibrium for the complete range of quota levels under the assumption that \(S < 3\).\(^{11}\)

**Proposition 1**

Assume \(S < 3\). The unique equilibrium of the pricing game depends on the level of the quota:

i) if \(q < 1 - S/2\), the domestic firm acts as a pure monopolist and the foreign one sell its quota at a maximum price, the market is not covered.

ii) if \(1 - S/2 \leq q < 1 - S/3\), the domestic firm covers the market but does not enter a price competition with the foreign firm.

iii) if \(1 - S/3 \leq q < \bar{q}\), an Edgeworth cycle appears where the domestic firm mixes between aggressive pricing and hiding behind the quota while the foreign firm plays a pure strategy.

iv) if \(q \geq \bar{q}\), firms play the Hotelling unit price.

**Proof** Observe that for any \(p_d\), \(D_f(p_d)\) is constant and then linear decreasing\(^{12}\), so that the profit function \(\Pi_f(p_d)\) is concave for any \(p_d\). Thus, whatever mixed strategy \(F_d\) the domestic firm might play, \(\Pi_f(F_d) = \int \Pi_f(p_d) dF_d(p_d)\) is concave and has a unique maximiser which means that in a Nash equilibrium, the foreign firm plays a pure strategy.

When the quota is very loose (case iv), the "classical" Hotelling equilibrium \((1,1)\) remains an equilibrium because the residual demand is too small. The analytical conditions derived from the best reply functions are \(\hat{p}_f < 1\) and \(1 < \hat{p}_d\). From the first, we get \([S - 1 + q](1 - q) < 1/2 \Rightarrow q > \overline{q} \equiv 1 - \frac{S - S^2/2}{2}\) (the other root is negative). From the second, we obtain \(q > 1/2\) which is satisfied by \(\overline{q}\) as \(S > 3/2\).

At the other extreme where \(q < 1 - S/2\) (case i), the domestic firm uses its dominant strategy \(S/2\)

\(^{11}\)This assumption is made in order to keep the exposition simple. The reader is referred to Boccard & Wauthy [97] for a treatment of equilibria for all values of \(S\).

\(^{12}\)Except for a case treated in footnote 8.
and the best reply of the foreign firm is then $S - q$; those prices form the unique Nash equilibrium which features an uncovered market because $D_d = S/2 < 1 - q$ and $D_f = q$. If the quota is only slightly larger (case ii), this kind of equilibrium where firms do not compete still prevails. The only difference is that the domestic firm plays $p_d = S - 1 + q$ against $S - q$ and the market is exactly covered. This behaviour is optimal for the domestic firm if $S - q < \hat{p}_f$ which leads to $q < 1 - S/3$. If $q \in [1 - S/3; \tilde{q}]$ (case iii), we have $1 < \hat{p}_f < S - q$ as on figure 3 which depicts a typical configuration of the best reply functions (in bold plain for $\psi_f$ and dashed for $\psi_d$). The best reply curves do not intersect, hence there exists no pure strategy equilibrium.

Figure 3

Still, the foreign firm plays a pure strategy in equilibrium. It must be $\hat{p}_f$ because it is the only one that enables the domestic firm to mix between $S - 1 + q$ and $1 + \hat{p}_f$ . Let $\mu$ be the weight put by the domestic firm strategy $F_d$ on $S - 1 + q$. The foreign profit against $F_d$ is $\Pi_f (F_d, p_f) = p_f \left(1 - \mu \right) q + \mu \tilde{x} \left(1 + \hat{p}_f/2, p_f\right)$. Solving for $\frac{\partial \Pi_f (F_d, p_f)}{\partial p_f} = 0$, we get the argmax of $\Pi_f (F_d, p_f)$ as a function $P(.)$ of $\mu$, we then solve $P(\mu) = \hat{p}_f$ to get $\mu = \frac{4q}{4q - 3 + 3 \hat{p}_f}$ which makes $\hat{p}_f$ a best reply for the foreign firm against $F_d$. ♦

Note first that by considering the complete range of possible values of the quota, we give a precise content to the idea of a quota "in the vicinity of the FTE" considered by Krishna [89]. More precisely, a mixed strategy equilibrium is found to exist only for a range of intermediate values of the quota (case iii), including the Free Trade equilibrium level.

Second, it is easy to relate the size of this interval $[1 - S/3; \tilde{q}]$ to $S$, the fundamental parameter of the model. As $\frac{\partial \pi}{\partial S} > 0$, the larger $S$, the larger the interval which supports a mixed strategy equilibrium. The reasons for this are quite intuitive: when $S$ is large, the profit levels at the Hotelling equilibrium are well below the monopoly profit levels. Indeed, the Hotelling prices do not depend on the common

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13 The technical condition is $\hat{p}_f < \frac{1 + \hat{p}_f}{2} \Leftrightarrow (S - 1 + q)(1 - q) < 2$ for otherwise $D_f \left(\hat{p}_f, p_d = \frac{1 + \hat{p}_f}{2}\right) = 0$ implying that $\Pi_f$ is locally increasing. This more complex case is analysed in Boccard N. & X. Wauthy (97). For the present paper, we can safely restrict $S$ to be lesser than 3 as will be clear in the proof of proposition 3.

14 Either because the reservation price is large or because the transportation cost is low i.e., products' differentiation is not
valuation of the good (see Lemma 1). As revealed by our previous analysis, the main implication of the quota is to allow the domestic firm to play, literally, along its monopolist profit curve. Therefore, the higher this curve, the greater the incentive to use the quota strategically. Since the security strategy basically allows the domestic producer to reach its monopoly profit curve, it has a great incentive to do so. This obviously implies that the range of quota levels which induce such strategies becomes larger.

A third observation is that a pure strategy equilibrium exists under highly restrictive quota levels (case ii when $S < 3$). This possibility was not considered by Krishna [89] and is indeed not relevant in her setting. This result is very specific to the Hotelling model and relies on localised competition which is embodied into the Hotelling framework: the quota weakens the incentives to compete in prices by allowing both firms to play along their respective local monopolists' curves. In equilibrium ii) the domestic firm has lost any incentive to compete in price and both firms name the highest price ensuring full market coverage, given the quota. Stated differently, a quota under horizontal differentiation essentially allows both firms to benefit from their local market advantages. In the limit, the domestic firm enjoys its full monopoly profits if both $q$ and $S$ are low (case i).

3) QUOTAS AND WELFARE

In this section we characterise the optimal level of the quota for a domestic government aiming at maximising domestic welfare. In order to achieve his goal, the government has to take three effects into account when choosing the level of the quota. First, there is the profit diversion effect, i.e. the part of the total welfare which is captured by the foreign producer in the presence of the quota. Second, there is the price differential effect: in our model, total welfare is maximised when prices are equal. Indeed, utility losses reflecting the fact that consumers are not able to buy their ideal product are minimised. Therefore, a quota inducing a price differential affects welfare negatively. Third, a quota affects welfare negatively if it prevents full market coverage.

The domestic welfare $W_d$ is obtained from the total welfare $W$ by subtracting the foreign profit. In the Hotelling model, $W$ is easily derived because, as long as the market is covered, it does not depend on the level of prices but only on the position of the marginal consumers (which depends in turn on price differentials only).

$$W(p_d, p_f) = \int_0^x (S - x)dx + \int_x^1 (S - 1 + x)dx = S - \frac{2x(p_d - p_f)^2 - 2x(p_d - p_f) + 1}{2}$$

It is immediate to see that $W$ is maximal when $\bar{x}(p_d, p_f) = 1/2$ i.e., for identical prices. The quota will therefore affect domestic welfare in two obvious ways, through foreign profits and through price differentials.
**Proposition 2**

The domestic government implements the minimal quota that ensures full market coverage.

**Proof** When the market is not covered (case \( i \) in Proposition 1), the domestic firm is a pure monopolist and the foreign firm is constrained by the quota, thus the total welfare reads:

\[
\forall q \in \left[ 0; 1 - S^2 \right], \quad W_d^i(q) = \int_0^1 (S - x)dx + \int_{1-q}^1 (S - 1 + x)dx = \frac{3}{8}S^2 + q(S - q/2)
\]

(E1)

As \( \Pi_f(q) = q(S - q) \) on this domain, domestic welfare is \( W_d^i(q) = \frac{3}{8}S^2 + \frac{q^2}{2} \) which is increasing in the quota. From this observation, we conclude that for low values of \( S \), the domestic government will not implement a fully projectionist policy, he will issue a quota that enables the foreign firm to serve the part of the market left uncovered by the domestic firm (who act as a monopolist on this range of quotas).

In case \( ii \) where \( q \in \left[ 1 - \frac{S}{2}; 1 - \frac{S}{3} \right] \), the marginal consumer is located at \( q \) and (E1) becomes

\[
W_d^{ii}(q) = S - \frac{1}{2} \left( 2(1-q)^2 - 2(1-q) + 1 \right) = \frac{1}{2} \left( 2q^2 - 2q + 1 \right).
\]

As the foreign profit is \( \Pi_f(q) = q(S - q) \), \( W_d^{ii}(q) = q(1 - S) + S - \frac{1}{2} \) which is decreasing with the quota. Hence the optimal quota over \( \left[ 0; 1 - \frac{S}{3} \right] \) is \( 1 - \frac{S}{2} \) which we call the "market-complement".

Case \( iii \) is the most complex because the domestic firm mixes between \( S - 1 + q \) and \( \hat{p}_f + \hat{p}_f \) with probabilities \( \bar{\mu} \) and \( 1 - \bar{\mu} \), the foreign firm sticks to \( \hat{p}_f \). Total welfare is thus the average formula

\[
W_d^{iii}(q) = \bar{\mu} W(S - 1 + q, \hat{p}_f) + (1 - \bar{\mu})W(\frac{1 + \hat{p}_f}{2}, \hat{p}_f) = \frac{2(6S - 3q - 6q^2 - 12q + 4q - 3)(4S + 3q - 2) - 9q(\hat{p}_f)^2}{4(4q - 3 + \hat{p}_f)}.
\]

A numerical analysis (available upon request) indicates that domestic welfare is strictly convex in the relevant domain, therefore it cannot be optimal for the government to set a quota in this area. Intuitively, this could have been expected since foreign profits tends to be higher in this area so that profit diversion affects domestic welfare negatively. Moreover prices are not equal so that total welfare must also be lower.

Lastly, when the Free Trade equilibrium prevails (case \( iv \)), total welfare is maximum because the marginal consumer is in the middle of the market. Since neither the foreign profit nor the total welfare depend on the quota, the domestic welfare is constant over \( \left[ \frac{S}{2}, 1 \right] \) with \( W_d^{iv} = S - \frac{3}{4} \). To find the overall optimal quota for the government, we have to compare the free trade solution (\( q = 1 \)) to either the "market-complement" one (\( q = 1 - \frac{S}{2} \)) or the complete protectionism (\( q = 0 \)) according to which applies.

When \( S > 2 \), the complete protectionism is optimal because \( W_d^{iv}(0) = S - 1/2 \). However, when \( S < 2 \), \( W_d^{iv}(1 - \frac{S}{2}) = \frac{1 - S + S^2}{2} > S - \frac{3}{4} = W_d^{iv} \), thus it is optimal to let the foreign firm cover the part of the market not served by the domestic monopolist. Note furthermore that this particular choice of quota is not followed by price competition as case \( i \) or \( ii \) applies in the equilibrium of the game. We have thus proven that the government always makes sure that the market will be covered but limits profit diversion to the minimum. ♦

The following comments are in order. It is only if the reservation price is low (\( 3/2 < S < 2 \)) that neither complete protectionism nor free trade are optimal for the government. Yet the analysis has been...
performed under zero production cost. It is easy to see that with symmetric constant marginal cost $c$, the relevant constellation would be $S - c \in \left[\frac{1}{2};2\right]$. Thus, the case for a restrictive quota depends in fact on the difference between the valuation of the product on the consumers' side in the domestic market and the production cost. Note also that the presence of a cost differential would not affect qualitatively our results. If the domestic producer faces a cost disadvantage, this reinforces the case for protectionism, other things being equal.

Last, we may identify the quota level that maximises the foreign firm's profit. Recall indeed that one of the key insight of Krishna [89] was to show that quotas could be voluntary precisely because they are instrumental in sustaining collusive outcomes. As shown in the next proposition, it is indeed optimal for the foreign firm to propose a VER, which should be located in the strict vicinity of the Free Trade equilibrium quantity.

**Proposition 3**

The optimal VER for the foreign producer is in the vicinity of the Free Trade Equilibrium quantity.

**Proof** The four kind of equilibria derived in proposition 1 enable to compute the foreign firm profit.

$$
\Pi_f(q) = \begin{cases} 
q(S - q) & \text{if } q \in [0;1 - \frac{S}{3}]
\end{cases}
$$

$$
\frac{2q \left(\sqrt{8(S-1+q)(1-q)}-1\right)}{4q^2-6q+3S(S-1-q)\sqrt{(1-q)}} & \text{if } q \in \left[1 - \frac{S}{3};q\right]
\end{cases}
$$

$$
\frac{2q \left(\sqrt{8(S-1+q)(1-q)}-1\right)}{4q^2-6q+3S(S-1-q)\sqrt{(1-q)}} & \text{if } q \in [q;1]
\end{cases}
$$

Observe that $S \geq \frac{1}{2} \Rightarrow S > \frac{S}{3} \Rightarrow \frac{S}{3} > 1 - \frac{S}{3} \Rightarrow \forall q \in \left[0;1 - \frac{S}{3}\right], S - 2q > 0$. This means that $\Pi_f(q)$ is increasing over $\left[0;1 - \frac{S}{3}\right]$. We check that over the interval $\left[1 - \frac{S}{3};q\right], \Pi_f(q)$ is strictly concave for any $S$ thus we may study $\frac{\partial \Pi_f}{\partial q}$ over $\left[1 - \frac{S}{3};q\right]$. The formula involves a 4th degree polynomial expression that can be solved analytically. The two real solutions are out of range but one of the complex solutions simplifies to real over the required interval. Over the range $\left[\frac{3}{2};3\right]$ we obtain $q(S) = 0.45 + 0.07\sqrt{S-1.5}$ which lies in the vicinity of Free Trade demand level $1/2$. Since profit is constant over $[q;1]$, we may conclude that $\Pi_f(q)$ reaches a maximum for a quota $q(S)$ interior to $\left[1 - \frac{S}{3};q\right]$.

Proposition 3 is quite intuitive. Choosing a VER in the vicinity of Free Trade allows the foreign producer to take advantage of the price effect associated with the quota, without penalising it too much in terms of potential sales. Note however that the profitability of the quota does not strictly depend on the equilibrium being in mixed strategies and may be desirable even if the equilibrium is in pure strategies.
4) Cournot competition

A standard criticism addressed to strategic trade policy models is that their conclusions are too dependent on the mode of competition, which is likely to be ignored by the government. Typically, in cases where an optimal policy under quantity competition is a subsidy to the home firm, it would be optimal to raise a tax under price competition. If the government lacks information about the mode of competition, policies based on a wrong belief may be harmful for the domestic welfare. This criticism seems particularly appropriate when the policy tool is a quota. Indeed, the results obtained up to now (non-existence of a pure strategy equilibrium, effective quota above the FTE level) are totally dependent on competition being in prices.

We now address this criticism directly and show, somewhat surprisingly, that in the present model, the quota is totally immune to it. To this end, we characterise a Cournot equilibrium in the Hotelling model. Lemma 2 shows that because the aggregate demand is inelastic, the Cournot equilibrium is not unique. However, the intuition underlying our characterisation is straightforward: for a pair of quantities to form a Cournot equilibrium, it must be that the corresponding market clearing prices are positive. As a direct consequence, the total quantity dumped in the market cannot exceed the market size. This defines as equilibrium candidates all pairs \( q_d + q_f = 1 \). Then we show that each firm sales must be "large enough" in a Cournot equilibrium and restrict accordingly the set of quantities which are Cournot equilibria. With this result in hand, we show that in order to maximise domestic welfare, the government will set an optimal quota which is identical to the one identified under price competition.

**Lemma 2 (Cournot competition)**

There exists a continuum of Cournot equilibria where the market is exactly covered the market clearing prices being \( p_d = S - q_d \) and \( p_f = S - q_f \).

**Proof** In the Cournot game, firms supply quantities \( q_i \) and \( q_j \) to an otherwise competitive market (i and j belong to \{d,f\}). If the quantities \( q_i \) and \( q_j \) do no cover the market, there is excess demand and prices increase until supply equals demand on each side of the market i.e., \( q_i = S - p_i \) and \( q_j = S - p_j \). Under the assumption \( S > 1 \), this situation is unstable since at least one firm has an incentive to increase its quantity above the complement of the other. If now the proposed quantities \( q_i \) and \( q_j \) exceed the market size, there is excess supply and at least one of the price, say \( p_i \), must be nil on this competitive market. Therefore firm i has a profitable deviation by offering a quantity slightly less than \( 1 - q_j \) to be on its monopoly profit curve.

The remaining candidates for a Cournot equilibrium are \( (q_i, 1 - q) \) with \( q \leq 1/2 \). The market clearing prices are \( S - q \) and \( S - 1 + q \). Without loss of generality, firm i offers \( q_i \) thus sells less than 1/2 in equilibrium. Hence, \( p_i \) cannot be nil because it would attract at least one half of the consumers, thereby implying an excess demand. Firm j cannot profitably deviate to a larger quantity than \( 1 - q \) because it would face a zero price (one price is nil and by the preceding argument, it must be its price).

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15 See for instance Maggi [96] for a critical discussion of this issue.

16 For exact market coverage, there is a continuum of prices which clear the market but the highest possible ones are the only ones that can appear in a subgame perfect equilibrium.
Firm $i$ may however profitably deviate to some $Q$ larger than $q$ but still less than $1/2$. Since there is excess supply, $p_j$ is nil, thus firm 1 sells all of $Q$ and the consumer located at $x = Q$ must be indifferent in equilibrium which means that $p_1 = 1 - 2Q$. The profit $Q(1 - 2Q)$ reaches a maximum of $\frac{1}{8}$ at $\frac{1}{4}$ to be compared with $q(S - q)$. Since $q \leq 1/2$, the only relevant root is $q^* = \frac{S - \sqrt{2S^2 - 1}}{2\sqrt{2}} > 0$. Finally, the Cournot equilibria feature exact market coverage $(q, 1 - q)$ for any $q > q^*$ if $S > 2$. For $S < 2$, the support of the continuum is possibly bounded by $S/2$.

**Proposition 4**

*The optimal quota is invariant to the mode of competition.*

**Proof** Lemma 2 shows that in our simple model, there exists a continuum of Cournot equilibria where the market is covered, the market clearing prices being $p_d = S - q_d$ and $p_f = S - q_f$. The inelasticity of the demand generates this continuum of equilibria and also the highest selling prices for firms. The intuition is clear: if quantities exceed the market size, at least one of the two prices must be equal to zero in order to clear the market. Therefore, the firm facing a zero price has an incentive to reduce its quantity to the complement of the other in order to benefit from a positive market clearing price. When quantities are small the market clearing prices are computed along each firm's local monopoly demand and each of them has an incentive to raise its quantity.

Moving from the Free Trade to government intervention reduces the continuum of equilibria since the strategy set for the foreign firm is now $[0;Q]$ where $Q$ is the quota set by the domestic government. Total welfare is still $W(p_d, p_f)$ but the prices are now the Cournot ones. Letting $q_f = 1 - q_d$ we obtain $W(S,q_d) = S - q_d^2 + q_d - \frac{1}{2}$. Since the foreigner's profit is $(1 - q_d)(S - 1 + q_d)$, the domestic welfare is $W_d(S,q_d) = q_d(S - 1) + \frac{1}{2}$. This expression is obviously increasing in $q_d$ as $S > 3/2$. It follows that in order to maximise domestic welfare, the government will choose the most restrictive quota compatible with market coverage. If $S > 2$, this amounts to choose complete protectionism ($Q = 0$) whereas if $S < 2$, the government must choose a restrictive quota allowing the foreign producer to sell the complement of the domestic monopolist ($Q = 1 - S/2$). In both cases, the continuum of Cournot equilibria is reduced to a single point the one obtained under price competition.

Under quantity competition, the quota changes the strategy set of the foreign firm but does not alter the nature of the competition. By contrast, the same quota has been shown to alter the nature of price competition. Thus, although the optimal quota is invariant to the mode of competition, it has drastically different implications depending on whether price or quantity competition prevails. Clearly, proposition 4 does not mean that the price competition analysis under quotas is redundant with respect to the quantity one. However, it conveys quite clearly the following message: the main impact of a quota in pricing games is to make equilibrium outcomes comparable to those prevailing under quantity competition. This appears clearly from the fact that in both cases, the relevant benchmark for the equilibrium payoffs is the same: the local monopoly payoffs. In Boccard and Wauthy [98], we study an industry where vertical differentiation prevails. Although the impact of the quota are quite different from the present paper, both approaches share the idea according to which, in a pricing game, the quota drives outcomes towards cournotian ones. This is not entirely surprising once it is recalled that the quota acts as a capacity constraint on the foreign producer.
It is indeed well known that introducing capacity constraints in a pricing game yields equilibrium payoffs exhibiting a cournotian flavour. This appears clearly from Kreps & Scheinkman [83] in the case of homogeneous products. The present analysis suggest that a similar result could obtain with differentiated products.

5) IMPORT QUOTA AND LOCATION CHOICE

We have shown in section 3 that the quota deeply alters price competition. As a consequence, we suspect that this alteration influences the choices of products’ attributes. There are two ways to address this issue. First, we may keep our linear transportation costs analysis and consider variable locations. It is well known that first firms invariably want to move towards the centre but also that once they are located to close from the centre, i.e. within $\left[\frac{1}{4}; \frac{3}{4}\right]$, no pure strategy equilibrium exists (for reasons completely different from those related to the quota; see d’Aspremont & al. [79] and Osborne & Pitchik [87]). Within this framework, it is possible to show that because of the quota, firms’ incentives differ. In particular, pure strategy equilibria may exist for inside locations and the tendency towards moving to the centre is weaker as shown by Wauthy [96]. Alternatively, we might modify our basic model by considering quadratic transportation costs. In this case, the existence of a pure strategy equilibrium is ensured under free trade, for all possible locations. It is also well-known that firms maximise differentiation in a subgame perfect equilibrium as we recall in Lemma 3 below. We then proceed to show that in the presence of the quota maximal differentiation cannot be sustained in equilibrium.

We quickly analyse the well known model of imperfect competition between two firms who first locate and then compete in price in a Hotelling setting. The only change with respect to the previous setting is that the domestic and foreign firms are located at $0 \leq x_d < x_f \leq 1$ and that consumers bear a quadratic transportation cost. The utility derived by a consumer located at $x$ in the interval $[0,1]$ is therefore

$$
\begin{cases}
S - (x_d - x)^2 - p_d & \text{if the product is bought at the foreign firm} \\
S - (x_f - x)^2 - p_f & \text{if the product is bought at the domestic firm}
\end{cases}
$$

Lemma 3 (The maximum differentiation principle)

If $S > 5/4$ and firms face no quantitative constraints, the only Nash equilibrium of the pricing game is $(1,1)$ and the market is covered. The location equilibrium is characterised by maximal differentiation.

Proof If prices are low then all consumers buy the good. The indifferent one is located at $\hat{x}(p_d,p_f) \equiv \frac{p_d - p_f}{2(x_d - x_f)} + \bar{x}$ the solution of $S - (x_d - x)^2 - p_d = S - (x_f - x)^2 - p_f$ where $\bar{x} \equiv \frac{x_d + x_f}{2}$. The foreign firm receives demand $\hat{x}(p_d,p_f)$ and the domestic firm receives demand $1 - \hat{x}(p_d,p_f)$. We assume that $S$ is large enough to ensure market coverage at the equilibrium.

The best reply of the domestic firm is $H_d(p_f) \equiv \frac{p_f}{2} + (x_d - x_f)(1 - \bar{x})$ while that of the foreign one is $H_f(p_d) \equiv \frac{p_d}{2} + (x_d - x_f)\bar{x}$. The equilibrium is $p_d^{FT} = \frac{2}{3}(x_d - x_f)(2 - \bar{x})$ and $p_f^{FT} = \frac{2}{3}(x_d - x_f)(1 + \bar{x})$. As $\hat{x}(p_d^{FT},p_f^{FT}) = \frac{4x_d + 3x_f}{5}$, the total cost of the indifferent consumer is $(x_d - \frac{4x_d + 3x_f}{5})^2 + p_d^{FT}$ while that of the right bound consumer is $(1 - x_d)^2 + p_d^{FT}$ and that of the left bound one is $(x_f)^2 + p_f^{FT}$. Those expressions are maximal when differentiation is maximum, thus $S > 5/4$ is sufficient to guarantee existence of the Hotelling
equilibrium for all possible pair of locations.\textsuperscript{17}

At the first stage where locations are chosen, payoffs are \( \Pi_d = \frac{2(x_d + x_f)(x_d(x_d - x_f))}{x_d^2} \) and \( \Pi_f = \frac{(2+x_d+x_f)(x_d(2x_d - x_f)(2x_f))}{18} \). The unconstrained best replies are \( L_d = \frac{4x_d}{3} \) and \( L_f = \frac{x_d^2 - 2}{3} \); they lead to optimal locations of -1/4 and 5/4. The equilibrium locations are therefore the boundaries of the market segment. The profits are then \( \Pi_d^{FT} = \frac{1}{2} = \Pi_f^{FT} \). \( \blacklozenge \)

We reintroduce an import quota and as before we keep the efficient rationing rule. The solution of \( D_f(p_d,p_f) = q \) is now \( p_f = p_d - 2(q - \bar{x})(x_d - x_f) \). Against a low price \( p_d \) the foreign firm is aggressive in order to gain market shares: its best reply is \( H_f(p_d) = \frac{p_d}{2} + (x_d - x_f)\bar{x} \) whenever the associated demand is below the quota. We are lead to solve \( D_f = \bar{x}(p_d,H_f(p_d)) \leq q \iff p_d \leq \hat{p}_d \equiv 2(x_d - x_f)(2q - \bar{x}) \). If \( p_d \) is larger then the optimal strategy for the foreign firm is to stick to the quota by playing \( p_f = p_d - 2(q - \bar{x})(x_d - x_f) \) thus the best reply is

\[
\psi_f(p_d) = \begin{cases} \frac{p_d}{2} + (x_d - x_f)\bar{x} & \text{if } p_d \leq \hat{p}_d \\ p_d - 2(q - \bar{x})(x_d - x_f) & \text{if } p_d > \hat{p}_d \end{cases}
\]

The domestic firm can either play the Free Trade best reply \( H_d(p_d) \) or take advantage of the quota to get a constant demand of \( 1 - q \). In the latter case the largest price \( p_d^* = S - (x_d - q)^2 \) consistent with this market share is optimal (assuming \( x_d \) is nearby 1 to guarantee that the demand is \( 1 - q \) for that price). The profits associated with each strategy are \( \frac{(p_d^* - \bar{x})^2}{2(x_d - x_f)(1 - \bar{x})} \) and \( (1-q)(S - (x_d - q)^2) \); they are equal for \( p_f = \hat{p}_f \equiv 4\sqrt{2(x_d - x_f)(1-q)(S - (x_d - q)^2)} - 2(x_d - x_f)(1 - \bar{x}) \). The discontinuous best reply function of the domestic firm is therefore

\[
\psi_f(p_d) = \begin{cases} S - (x_d - q)^2 & \text{if } p_f \leq \hat{p}_f \\ \frac{p_d^*}{2} + (x_d - x_f)(1 - \bar{x}) & \text{if } p_f > \hat{p}_f \end{cases}
\]

**Proposition 5**

There exist no location equilibrium with maximum differentiation in the presence of a quota lesser than \( \bar{q}(S,0,1) = 0.9 - S/2 + 1.8\sqrt{S} \).

**Proof** For a loose quota, the classical Hotelling pair \( (p_d^{FT},p_f^{FT}) \) is the equilibrium because the residual demand is too small. The conditions derived from the best reply functions are \( \hat{p}_f < p_f^{FT} \) and \( p_d^{FT} < \hat{p}_d \). From the first inequality, we get \( q > \bar{q}(S,x_d,x_f)^{18} \) where the benchmark decreases with differentiation. From the second inequality, we obtain \( q > \frac{1+\bar{x}}{3} \) which is satisfied as \( \bar{q}(S,x_d,x_f) > \frac{1+\bar{x}}{3} \) is always true.

When \( q < \bar{q}(S,x_d,x_f) \) the equilibrium sees the foreign firm playing the pure strategy \( \hat{p}_f \) while the domestic mixes between the security one \( p_d^* \) and the Hotelling best reply \( H_d(\hat{p}_f) \).

\textsuperscript{17} If firms play prices such that the market is uncovered then each is a local monopoly. It then happens that because \( S > 5/4 \) the pair of monopoly prices they wish to play leads to market covering.

\textsuperscript{18} The formula is available upon request from the authors.
For any \( q < \frac{S}{2} \), we have \( q < \frac{S}{2} \) (S,d,1) and \( q < \frac{S}{2} \) (S,0,x_f) thus when a firm moves toward the centre the equilibrium is of the second type. The equilibrium payoff is \( \Pi_d(x_d) = (1 - q)(S - (x_d - q)^2) \) for the domestic firm; it will therefore move toward the centre of the market and choose \( x_d^* = 1 - \frac{1 - q}{2} = \frac{1 + q}{2} \) in order to minimise transportation cost over its protected market share. As in proposition 1 the foreign profit is \( \Pi_f(p_f) = p_f\left((1 - \mu)q + \mu\left(1 - \hat{x}_d(p_f) - p_f \right)\right) \) where \( \mu \) is the weight put by the domestic firm on \( p_d \). Solving for \( \frac{\partial \Pi_f(p_f)}{\partial p_f} = 0 \) at \( p_f = \hat{p}_f \) yields an optimal \( \hat{\mu} \). The equilibrium payoff can then be expressed as a function of location: \( \Pi_f(x_f) = \frac{2q\hat{p}_f}{3p_f - (x_d - x_f)(2 - 4q + x_d + x_f)} \). We are then able to study the best reply to \( x_d^* = \frac{1 + q}{2} \) provided that the left boundary consumer still buys from the foreign firm. A numerical computation (not a simulation) shows that \( x_f^* = 2q - 1 \) for \( q > 55\% \) (for \( S \) in the range \( \frac{3}{2};3 \)) and zero otherwise. Since the domain of existence of the mixed strategy equilibrium is for intermediate quotas\(^{19} \), we may conclude that a quota just below \( \frac{S}{2} \) (0.9 - \( S/2 + 1.8\sqrt{S} \) (which ranges from 65% to 85% for \( S \) in the range \( \frac{3}{2};3 \))) would lead both firm to move towards the centre of the market at \( x_d^* = \frac{1 + q}{2} \) and \( x_f^* = 2q - 1 \). ♦

Note that this proposition does not provide a complete analysis of location choices. We have only shown that, contrarily to free trade, maximum differentiation is an unstable configuration. Moreover, this holds for apparently innocuous quotas: typically, quotas less 65% or 85% (depending on \( S \)) have such implications. These levels are much above than the Free trade level of sales for the foreign firm (50%). We are not claiming either that a minimal differentiation will occur as in Boccard & Wauthy [98] who study vertical differentiation. Yet the localisation by the domestic firm in the middle of its protected market is likely to be a robust tendency; this would also exclude minimal differentiation. Accordingly, we can safely conjecture that the quota will induce intermediate differentiation configurations. In any case, it is worth recalling that under free trade firms were inclined to avoid moving to the centre in order to relax competition. It should be clear by now that if firms move towards the centre in the presence of the quota, it is precisely because the quota offers an alternative way of relaxing competition.

Notice that the optimal quota from the domestic point of view remains protectionism as the local monopoly will locate in the middle of the market segment to minimise transportation cost.

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\(^{19}\) A constrained pure strategy equilibrium similar to case ii of proposition 1 holds otherwise.
6) CONCLUSION

Very few papers developed the seminal paper of Krishna [89] further on. As a consequence, very little is known about the implications of quotas in pricing games, beyond the fact that they may act as facilitating practices. In this paper, we have studied the implication of import restraints in an address-model of horizontal differentiation. By using an explicit model, we are able to study the impact of the quota in full details. In this setting, the quota dramatically relaxes price competition to the benefits of both firms, and this for a very large domain of the parameters. As shown in the previous section this effect weakens the incentives to differentiate product attributes.

A quota in the vicinity of Free Trade is optimal for the foreign firm but is particularly damaging from a domestic welfare point of view. The optimal policy is therefore to implement a restrictive quota which guarantees full market coverage. Finally, we show that the optimal quota is invariant to the mode of competition (Bertrand vs. Cournot).

Our findings are admittedly specific to the particular model we have studied, however several generalisations can be considered. Firstly, we use the efficient rationing rule. In this respect, it must be noted that this form of rationing is the most favourable for the domestic producer. In this sense, any other rationing rule would make deviations less profitable, thereby reducing the domain in which the quota leads to a mixed strategy equilibrium. On the other hand, under other rationing rules, some of the rationed consumers may not consume at equilibrium which has a negative impact on Domestic Welfare.

Note finally that our analysis has been confined to the case of horizontal differentiation. In Boccard & Wauthy [98] we study a similar problem under vertical differentiation. In both settings the impact of the quota is to turn price competition into a quantity-like competition. Nevertheless the implications for the choice of products characteristics are different. In the vertical case the quota induces minimal differentiation while we have just shown that neither maximal nor minimal differentiation should obtain in the Hotelling model. Investigating the exact equilibrium of the location game is left for future research.

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